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Barrier lake formation due to landslide impacting a river

- A numerical study using a double layer-averaged two-phase flow model

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The supplementary materials include a) model closures for the governing equations of the proposed double layer-averaged two-phase flow mode (Text S1); b) numerical algorithm for solving the model (Text S2); c) summary of numerical cases and the results (Table S1 for Series 3 and Table S2 for Series 4); d) summary of computed critical indexes for barrier lake formation for Series 3 and 4 (Table S3); e) a video showing the formation process of barrier lake formation due to landslide impacting a river, in relation to Case 4-1 (Video S1).

Text S1 Model closures

S1.1. Sediment exchange with the bed

Estimation of sediment exchange with the bed is one of the key components of computational models for shallow water-sediment flows over erodible beds. Such exchanges are important for both fluvial sediment-laden flows and general geophysical mass flows such as landslides, debris flows, and avalanches. In fluvial hydraulics, two distinct mechanisms are usually involved in the mass exchange with the bed: upward bed sediment entrainment due to interphase and inter-grain size interactions and downward sediment deposition primarily as the result of gravitational action [1]. However, in geophysical mass flows, understanding of the underlying physical processes remains unclear [2-3]. This is despite the large number of entrainment rate equations describing the erosion/deposition rate of the basal surface that have been proposed to estimate the mass exchange with the bed induced by geophysical mass flows [3-5]. Existing formulations suffer from several shortcomings. Notably, most existing bed-entrainment rate equations do not consider the effect of particle size. This is questionable from a physical perspective because fine grains are easier to erode than large blocks [6]. Given these observations, Li et al. [7] tentatively employed the widely used closure model in fluvial hydraulics to estimate the mass exchange between the debris flow and the bed, and found that the model performs significantly better than an existing entrainment rate equation derived from analytical models [5]. Therefore, the closure model originated from fluvial hydraulics is adopted in this study. Accordingly, the size-specific net flux of sediment exchange with the bed is

$$F_k = E_k - D_k \tag{S1}$$

where k refers to a given particle size; E_k is the size-specific sediment entrainment flux; and D_k is the size-specific sediment deposition flux. By summation, $E_T = \sum E_k$ is the total sediment entrainment flux and $D_T = \sum D_k$ is the total sediment deposition flux. In practice, the deposition flux can be determined using the local near-bed sediment concentration and the hindered settling velocity. One of the most widely used approaches to specifying sediment entrainment flux is based on the assumption that entrainment always occurs at the same rate as under the capacity regime. At capacity conditions, the entrainment flux is equal to the deposition flux. Therefore, bed sediment entrainment flux can be computed by using the near-bed sediment concentration at capacity and the settling velocity.

Accordingly, entrainment and deposition fluxes are estimated from

$$E_k = \omega_k c_{ek}, \quad D = \omega_k c_k (1-c_k)^{m_k} (1-c_T)^{m_0} \cos\psi_m \tag{S2a, b}$$

where ω_k is the settling velocity of the k th grain size, calculated using Zhang's formula [8] as,

$$\omega_k = \sqrt{(13.95\frac{\nu_{\mu f}}{d_k})^2 + 1.09 s g_{\psi_m} d_k} - 13.95\frac{\nu_{\mu f}}{d_k} \tag{S3}$$

where $\nu_{\mu f}$ is kinematic viscosity of the water phase; $s = (\rho_s/\rho_w - 1)$; and hindered sediment settling is taken into account in deposition flux by incorporating the effects from the water phase using the Richardson and Zaki [9] formula and also other constituents of the sediment phases, where $m_k = 4.45 R_{pk}^{-0.1}$, in which $R_{pk} = \omega_k d_k / \nu_{\mu f}$ is the size-specific particle Reynolds number, $m_0 = 4.45 R_{p0}^{-0.1}$, R_{p0} is determined by the mean sediment diameter d_m, where $d_m = \sum (c_k d_k)/c_T$. The size-specific sediment concentration c_{ek} at capacity is determined as,

$$c_{ek} = A_k q_k / (h_m U_m) \tag{S4a}$$

where q_k is the size-specific transport rate at capacity regime, calculated using the Wu formula [1]; $U_m = \sqrt{u_m^2 + v_m^2}$ is the depth-averaged velocity of the lower water-sediment mixture flow layer; and A_k is the areal exposure fraction of the k th sediment on the bed surface given by Parker [10-11] as below,

$$A_k = \frac{f_{ak} / \sqrt{d_k}}{\sum (f_{ak} / \sqrt{d_k})} \quad . \tag{S4b}$$

The sediment transport rate of any size fraction is determined as [1],

$$\frac{q_k}{\phi_k \sqrt{s g_{\psi_m} d_k^3}} = 0.0053 \left[(\frac{n'}{n_b})^{1.5} \frac{\tau_b}{\tau_{ck}} - 1 \right]^{2.2} + 0.0000262 \left[(\frac{\tau}{\tau_{ck}} - 1) \frac{U_m}{\omega_k} \right]^{1.74} , \tag{S4c}$$

where ϕ_k is the modification coefficient, which covers a wide range in existing computational models. In the present study, $\phi_k =$ 1. Therefore, parameter adjustment is not required, and the uncertainty is reduced. $n' = d_{50}^{1/6} / 20$ is the Manning roughness coefficient corresponding to grain resistance; τ is the shear stress at channel cross-section and is estimated as $\rho_m RJ$, where R is the hydraulic radius of the channel cross-section and J is the friction slope; τ_{ck} is the critical shear stress for incipient motion of bed material, approximated by $\tau_{ck} = 0.03 \alpha_k (\rho_s - \rho_w) g_{\psi_m} d_k$ with α_k a correction factor accounting for the hiding and exposure mechanisms in multi-grain sized sediments [1]. In a sense, the sediment transport rate is essentially a function of shear stresses acting on the bed, including the bed shear stress and the critical shear stress. In fact, this is consistent with conventional ideas in existing equations used for bed entrainment rate estimation [3, 5] in that the entrainment of bed sediment results from stresses exerted on the basal surface. Note that Eq. (S2a) is applicable when there is sufficient sediment supply from the bed. Otherwise, the sediment entrainment flux vanishes where the bed is made of rigid materials (e.g., steel or concrete) and is locally non-erodible. The following relation is employed to evaluate f_{Ik} [12-13],

$$f_{Ik} = \begin{cases} f_{sk} & \partial\xi/\partial t \le 0 \\ \varphi c_k / c_T + (1-\varphi) f_{ak} & \partial\xi/\partial t > 0 \end{cases} \tag{S5}$$

where f_{sk} is the fraction of the k th size sediment in the substrate layer; and φ is the empirical weighting parameter. When bed degrades (i.e., $\partial\xi/\partial t \le 0$), f_{Ik} is directly determined by the fraction in the original substrate specified a priori, f_{sk}. Yet, when the bed aggrades (i.e., $\partial\xi/\partial t > 0$), the determination of f_{Ik} relies on the average fraction in the active layer (i.e., f_{ak}) and the sediment concentration. This is reasonable because sediments within the flow would affect the bed grain-size stratigraphy. As shown in the equations above, the effect of grain size is properly incorporated into the setting velocity (Eq. S3), the size-specific areal exposure fraction (Eq. S4b) and the sediment transport rate of each size group (Eq. S4c). Generally, finer grains are easier to erode than coarser grains.

S1.2. Shear stresses

At present, no universal closure models are available for representing bottom shear stresses for the clear-water flow layer and the lower water-sediment mixture layer. Inevitably, empiricism is introduced with regard to shear stress estimation (and common to all shallow water-sediment flow models).

We follow conventional practice in two-phase flow modeling by dividing the total bed shear stresses for the water-sediment mixture into respective bed shear stresses exerted on the water and sediment phases [14-16],

$$\tau_b = \tau_{fb} + \sum(\tau_{s_k b}) \tag{S6}$$

The solid resistance in the water-sediment mixture flow layer is determined by the Coulomb friction law [17], which expresses the collinearity of shear stress and normal stress through a friction coefficient $\tan\phi_{bed}$. Following this practice, the solid resistance components are given as follows,

$$\tau_{s_k bx} = (\rho_s - \rho_w) g_{\psi_m} h_{sk} \tan\phi_{bed} \frac{u_{sk}}{\sqrt{u_{sk}^2 + v_{sk}^2}} \tag{S7a}$$

$$\tau_{s_k by} = (\rho_s - \rho_w) g_{\psi_m} h_{sk} \tan\phi_{bed} \frac{v_{sk}}{\sqrt{u_{sk}^2 + v_{sk}^2}} \tag{S7b}$$

Separately, the fluid resistance components in the water-sediment mixture flow layer are estimated using Manning's equation,

$$\tau_{fbx} = \rho_w \frac{gn^2}{(h_f \cos\psi_m)^{1/3}} \frac{u_f |u_f|}{\cos\psi_m} \tag{S8a}$$

$$\tau_{fby} = \rho_w \frac{gn^2}{(h_f \cos\psi_m)^{1/3}} \frac{v_f |v_f|}{\cos\psi_m} \tag{S8b}$$

Similarly, the bottom shear stress components for the clear-water flow layer is estimated using Manning's equation [18],

$$\tau_{wx} = \rho_w \frac{gn^2}{(h_w \cos\psi_w)^{1/3}} \frac{(u_w - u_m)\bar{U}_{wm}}{\cos\psi_w} \tag{S9a}$$

$$\tau_{wy} = \rho_w \frac{gn^2}{(h_w \cos\psi_w)^{1/3}} \frac{(v_w - v_m)\bar{U}_{wm}}{\cos\psi_w} \tag{S9b}$$

where n is the Manning roughness parameter; $\bar{U}_{wm} = \sqrt{(u_w - u_m)^2 + (v_w - v_m)^2}$ is the resultant velocity difference between the two flow layers. Note that $n = n_w$ where n_w is the roughness at the interface between the two flow layers, when the water-sediment mixture layer flows underneath the clear-water flow layer. In other cases, $n = n_b$, where n_b is the bed roughness.

S1.3. Interaction force

The interphase interaction force mainly comprises the drag force, virtual (added) mass force, and lift force. In general, the latter two forces are neglected in shallow water-sediment flow models [19-20], except for a two-phase flow model by Pudasaini [15], which specifically includes the virtual mass force. However, the effect of the virtual mass force is negligible according to the numerical results obtained by Pudasaini [15]. The interphase drag force components F_{Dkx} and F_{Dky} are expressed:

$$F_{Dkx} = \rho_w D_{rk} h_m (u_f - u_{sk}) \tag{S10a}$$

$$F_{Dky} = \rho_w D_{rk} h_m (v_f - v_{sk}) \tag{S10b}$$

where D_{rk} is the drag function and can be determined on the basis of drag correlation, following Gidaspow [21]

$$D_{rk} = \begin{cases} 150 \dfrac{c_k^2 \nu_{\mu f}}{(1 - \sum c_k) d_k^2} + \dfrac{7}{4} \dfrac{c_k}{d_k} \sqrt{(u_f - u_{sk})^2 + (v_f - v_{sk})^2} & if\ c_k > 0.2 \\ \dfrac{3}{4} c_d(\mathrm{Re}_k) \dfrac{(1 - \sum c_k) c_k}{d_k} (1 - \sum c_k)^{-2.65} \sqrt{(u_f - u_{sk})^2 + (v_f - v_{sk})^2} & if\ c_k \le 0.2 \end{cases} \tag{S11a}$$

where the drag coefficient $c_d(\mathrm{Re}_k)$ is given by

$$c_d = \begin{cases} \dfrac{24}{\mathrm{Re}_k}(1.0 + 0.15\mathrm{Re}_k^{\ 0.687}) & if\ \mathrm{Re}_k < 1000 \\ 0.44 & if\ \mathrm{Re}_k \geq 1000 \end{cases} \tag{S11b}$$

in which $\mathrm{Re}_k = c_f \sqrt{(u_f - u_{sk})^2 + (v_f - v_{sk})^2}\, d_k \big/ \nu_{\mu f}$ is the size-specific Reynolds number of the flow, and $\nu_{\mu f}$ is the kinematic viscosity of the fluid phase.

According to Gray and Chugunov [22], the inter-grain size interaction drag force is made up from a linear velocity-dependent drag force, a grain-grain surface interaction force, and a remixing force. By depth-averaging, the size-specific interaction drag components $F_{s-s_k x}$ and $F_{s-s_k y}$, are formulated as follows:

$$F_{s-s_k x} = \int_{z_b}^{\eta} f_{s-s_k x} dz = \frac{1}{2} c_T \rho_m g_{\psi_m} h_m^2 \frac{\partial}{\partial x}(\frac{c_k}{c_T}) - \rho_s \frac{c_k}{c_T} c_{sd} (u_{sk} - \overline{u}_s) h_m - \rho_s \nu_d h_m \frac{\partial}{\partial x}(\frac{c_k}{c_T}) \tag{S12a}$$

$$F_{s-s_k y} = \int_{z_b}^{\eta} f_{s-s_k y} dz = \frac{1}{2} c_T \rho_m g_{\psi_m} h_m^2 \frac{\partial}{\partial y}(\frac{c_k}{c_T}) - \rho_s \frac{c_k}{c_T} c_{sd} (v_{sk} - \overline{v}_s) h_m - \rho_s \nu_d h_m \frac{\partial}{\partial y}(\frac{c_k}{c_T}) \tag{S12b}$$

where $\overline{u}_s = \sum (c_k u_{sk}) \big/ c_T$, $\overline{v}_s = \sum (c_k v_{sk}) \big/ c_T$, c_{sd} is the linear drag coefficient, and ν_d is the linear diffusivity coefficient. In the present study, $c_{sd} =$ 6.3 s^{-1} and $\nu_d = 1.26 \times 10^{-5}$ m^2 s^{-2} following Hill and Tan [23].

S1.4. Water entrainment

The mass flux of water entrainment E_w represents the mixing of the water-sediment mixture with the clear water across the interface of the two moving layers. A slightly adapted version of the relationship initially proposed for turbidity currents [24] is tentatively used for determining E_w as follows,

$$E_w = e_w \overline{U}_{wm} \tag{S13}$$

where the water entrainment coefficient e_w is calculated empirically using the Richardson number $Ri = g' h_s / \bar{U}_{wm}^2$ and the submerged gravitational acceleration $g' = s g_{\psi_m} c_T$ with specific gravity of sediment $s = \rho_s / \rho_w - 1$, as

$$e_w = \frac{0.00153}{0.0204 + Ri} \quad . \qquad \text{(S14)}$$

Text S2 Numerical algorithm

Following the numerical strategy proposed by Cao et al. [25], Eqs. (1-12) can be decomposed into two reduced-order systems representing the clear-water flow layer and the water-sediment mixture flow layer, whereas the bed deformation equation (Eq. 13) and the active layer equation (Eq. 14) are solved separately from the remaining equations. Moreover, only two of the three governing equation systems for the water-sediment mixture (Eqs. 4-6), the sediment phase (Eqs. 7-9), and the water phase (Eqs. 10-12) are independent and can in principle be used in formulating the mathematical model for the lower water-sediment mixture flow layer. As suggested by Li et al. [7, 26-27], the governing equation system for the water-sediment mixture flow layer is composed of the equations for the water-sediment mixture and the sediment phase because this system is hyperbolic, characterized by the straightforward derivation of the real and distinct eigenvalues. Specifically, in this study, Eqs. (1-9) and (13-14) are used to formulate the double layer-averaged two-phase flow model.

To expedite numerical solution using conservative variables, it is advisable to recast Eqs. (1-9) so that the densities do not appear on the left hand side (LHS) of the equations. Thus, the governing equations of the present double layer-averaged two-phase flow model are written in a standard, well-structured conservative form.

$$\frac{\partial \mathbf{T}}{\partial t}+\frac{\partial \mathbf{E}}{\partial x}+\frac{\partial \mathbf{F}}{\partial y}=\mathbf{R}_b+\mathbf{R}_f+\mathbf{R}_e \tag{S15}$$

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{G}}{\partial x}+\frac{\partial \mathbf{H}}{\partial y}=\mathbf{S}_b+\mathbf{S}_f+\mathbf{S}_e \tag{S16}$$

$$\mathbf{T}=\begin{bmatrix}\eta\\ q_{wx}\\ q_{wy}\end{bmatrix}=\begin{bmatrix}\eta\\ h_w u_w\\ h_w v_w\end{bmatrix} \tag{S17a}$$

$$\mathbf{E}=\begin{bmatrix}h_w u_w\\ h_w u_w^2+\frac{1}{2}g_{\psi_w}(\eta^2-2\eta\eta_s)\\ h_w u_w v_w\end{bmatrix},\quad \mathbf{F}=\begin{bmatrix}h_w v_w\\ h_w u_w v_w\\ h_w v_w^2+\frac{1}{2}g_{\psi_w}(\eta^2-2\eta\eta_s)\end{bmatrix} \tag{S17b, c}$$

$$\mathbf{R}_b=\begin{bmatrix}0\\ -g_{\psi_w}\eta\frac{\partial \eta_s}{\partial x}\\ -g_{\psi_w}\eta\frac{\partial \eta_s}{\partial y}\end{bmatrix},\quad \mathbf{R}_f=\begin{bmatrix}0\\ -\frac{\tau_{wx}}{\rho_w}-E_w u_w\\ -\frac{\tau_{wy}}{\rho_w}-E_w v_w\end{bmatrix},\quad \mathbf{R}_e=\begin{bmatrix}-E_w+\frac{\partial \eta_s}{\partial t}\\ 0\\ 0\end{bmatrix} \tag{S17d, e, f}$$

$$\mathbf{U}=\begin{bmatrix}\eta_m\\ q_{mx}\\ q_{my}\\ h_{sk}\\ q_{skx}\\ q_{sky}\end{bmatrix}=\begin{bmatrix}\eta_m\\ h_m u_m\\ h_m v_m\\ h_{sk}\\ h_{sk}u_{sk}\\ h_{sk}v_{sk}\end{bmatrix} \tag{S18a}$$

$$\mathbf{G}=\begin{bmatrix}h_m u_m\\ h_m u_m^2+0.5g_{\psi_m}(\eta_m^2-2\eta_m z_b)\\ h_m u_m v_m\\ h_{sk}u_{sk}\\ h_{sk}u_{sk}^2+0.5g_{\psi_m}(\rho_m/\rho_s)c_k h_m^2\\ h_{sk}u_{sk}v_{sk}\end{bmatrix},\quad \mathbf{H}=\begin{bmatrix}h_m v_m\\ h_m u_m v_m\\ h_m v_m^2+0.5g_{\psi_m}(\eta_m^2-2\eta_m z_b)\\ h_{sk}v_{sk}\\ h_{sk}u_{sk}v_{sk}\\ 0.5g_{\psi_m}(\rho_m/\rho_s)c_k h_m^2\end{bmatrix} \tag{S18b, c}$$

$$\mathbf{S}_b = \begin{bmatrix} 0 \\ -g_{\psi_m}\eta_m \dfrac{\partial z_b}{\partial x} \\ -g_{\psi_m}\eta_m \dfrac{\partial z_b}{\partial y} \\ 0 \\ -(\dfrac{\rho_m}{\rho_s})g_{\psi_m}\eta_m \dfrac{\partial z_b}{\partial x} \\ -(\dfrac{\rho_m}{\rho_s})g_{\psi_m}\eta_m \dfrac{\partial z_b}{\partial y} \end{bmatrix}, \quad \mathbf{S}_f = \begin{bmatrix} N_m \\ N_{mx} \\ N_{my} \\ F_k \\ N_{skx} \\ N_{sky} \end{bmatrix}, \quad \mathbf{S}_e = \begin{bmatrix} E_w \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(S18d, e, f)}$$

$$N_m = \frac{\rho_s - \rho_w}{\rho_w} \sum \left([\frac{\partial h_m c_k (u_m - u_{sk})}{\partial x}] + [\frac{\partial h_m c_k (v_m - v_{sk})}{\partial y}] \right) \qquad \text{(S18g)}$$

$$\begin{aligned} N_{mx} = & \frac{\tau_{wx} - \tau_{bx}}{\rho_m} - \frac{(\rho_0 - \rho_m)(E-D)u_m}{(1-p)\rho_m} + \frac{(\rho_s - \rho_w)u_m E_w c_T}{\rho_m} \\ & + \frac{\rho_w E_w u_w}{\rho_m} - \frac{(\rho_s - \rho_w)g_{\psi_m} h_m^2}{2\rho_m}\frac{\partial c_T}{\partial x} - \frac{\rho_w}{\rho_m} g_{\psi_w} h_m \frac{\partial h_w}{\partial x} \\ & - \frac{1}{\rho_m}\frac{\partial}{\partial x} h_m \sum [\rho_s c_k i_{s_k x}(i_{s_k x} - i_{fx})] - \frac{1}{\rho_m}\frac{\partial}{\partial y} h_m \sum [\rho_s c_k i_{s_k x}(i_{s_k y} - i_{fy})] \end{aligned} \qquad \text{(S18h)}$$

$$\begin{aligned} N_{my} = & \frac{\tau_{wy} - \tau_{by}}{\rho_m} - \frac{(\rho_0 - \rho_m)(E-D)v_m}{(1-p)\rho_m} + \frac{(\rho_s - \rho_w)v_m E_w c_T}{\rho_m} \\ & + \frac{\rho_w E_w v_w}{\rho_m} - \frac{(\rho_s - \rho_w)g_{\psi_m} h_m^2}{2\rho_m}\frac{\partial c_T}{\partial y} - \frac{\rho_w}{\rho_m} g_{\psi_w} h_m \frac{\partial h_w}{\partial y} \\ & - \frac{1}{\rho_m}\frac{\partial}{\partial y} h_m \sum [\rho_s c_k i_{s_k y}(i_{s_k y} - i_{fy})] - \frac{1}{\rho_m}\frac{\partial}{\partial x} h_m \sum [\rho_s c_k i_{s_k y}(i_{s_k x} - i_{fx})] \end{aligned} \qquad \text{(S18i)}$$

$$N_{s_k x} = \frac{\tau_{wx} c_k - \tau_{s_k bx}}{\rho_s} + \frac{F_{s_k fx} + F_{s-s_k x}}{\rho_s} + \frac{1}{2}\frac{\rho_w}{\rho_s} g_{\psi_m} h_m^2 \frac{\partial c_k}{\partial x} - \frac{\rho_w}{\rho_s} g_{\psi_w} h_{sk} \frac{\partial h_w}{\partial x} \qquad \text{(S18j)}$$

$$N_{s_k y} = \frac{\tau_{wy} c_k - \tau_{s_k by}}{\rho_s} + \frac{F_{s_k fy} + F_{s-s_k y}}{\rho_s} + \frac{1}{2}\frac{\rho_w}{\rho_s} g_{\psi_m} h_m^2 \frac{\partial c_k}{\partial y} - \frac{\rho_w}{\rho_s} g_{\psi_w} h_{sk} \frac{\partial h_w}{\partial y} \qquad \text{(S18k)}$$

where $\eta = \eta_s + h_w$ is the water surface elevation above the bed; $\mathbf{T}$ and $\mathbf{U}$ are vectors of conserved variables; $\mathbf{E}$, $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{H}$ are vectors of flux variables. For the lower water-sediment mixture flow layer, $\mathbf{S}_b$ denotes the vector of the bed-slope source terms, $\mathbf{S}_f$ is the vector of the bed-friction source terms and other terms related to the impacts of sediment transport and water entrainment, and $\mathbf{S}_e$ is the vector of the water-entrainment source terms. Similarly, for the clear-water flow layer, $\mathbf{R}_b$ is the vector of interface-slope source terms, $\mathbf{R}_f$ is the vector of friction source terms and other terms related to the impacts of water entrainment, $\mathbf{R}_e$ is the vector of the water-entrainment source terms and the interface-elevation variation terms; q_{wx}, q_{wy} are the conservative variables in Eq. (S17a); and q_{mx}, q_{my}, q_{skx}, q_{sky} are the conservative variables in Eq. (S18a).

Eqs. (S15) and (S16) constitute two non-homogeneous hyperbolic systems that can be solved separately and synchronously using one of a hierarchy of numerical algorithms that can capture shock waves and contact discontinuities properly [28]. Taking Eq. (S16) as an example, an adapted version of the numerical algorithm in Cao et al. [25] is employed. Briefly, an explicit finite volume discretization [28] is applied along with a second-order Runge-Kutta (RK) method used for the source terms. In principle, the present model is well-balanced because the interlayer interactions [primarily the sixth terms on the right hand side (RHS) of Eqs. (S18h) and (S18i)] play secondary roles, and are negligible compared with inertia and gravitation [29].

$$\mathbf{U}_{i,j}^{*} = \mathbf{U}_{i,j}^{m} - \frac{\Delta t(\mathbf{G}_{i+1/2,j} - \mathbf{G}_{i-1/2,j})^{m}}{\Delta x} - \frac{\Delta t(\mathbf{H}_{i,j+1/2} - \mathbf{H}_{i,j-1/2})^{m}}{\Delta y} + \Delta t \overline{\mathbf{S}}_{bi,j}^{m} \tag{S19a}$$

$$\mathbf{U}_{i,j}^{m+1} = \mathbf{U}_{i,j}^{*} + \Delta t(\mathbf{S}_{f\,i,j}^{RK} + \mathbf{S}_{ei,j}^{RK}) \tag{S19b}$$

where Δt is the time step; Δx and Δy are the spatial increments; the subscripts i and j denote the spatial node indices; the superscript m denotes the time step index; the superscript * indicates the state after calculating the variables in Eq. (S19a); $\mathbf{G}_{i+1/2,j}$, $\mathbf{G}_{i-1/2,j}$, $\mathbf{H}_{i,j+1/2}$ and $\mathbf{H}_{i,j-1/2}$ represent the inter-cell numerical fluxes.

The bed slope source term $\overline{\mathbf{S}}_{bi,j}$ is discretized with a centered difference scheme [30-31] given that it is well-balanced with the flux gradients. The source terms $\mathbf{S}_f$ and $\mathbf{S}_e$ are computed by the second-order Runge-Kutta method as follows,

$$\mathbf{S}^{RK} = [\mathbf{S}(\mathbf{U}_{i,j}^{*1}) + \mathbf{S}(\mathbf{U}_{i,j}^{*2})]/2 \quad , \tag{S20}$$

with $\mathbf{U}_{i,j}^{*1} = \mathbf{U}_{i,j}^{*}$, $\mathbf{U}_{i,j}^{*2} = \mathbf{U}_{i,j}^{*1} + \Delta t\mathbf{S}(\mathbf{U}_{i,j}^{*1})$.

The bed deformation and bed surface material composition are updated by the discretized version of Eqs. (13) and (14) respectively

$$z_{b\,i}^{m+1} = z_{b\,i}^{m} - \Delta t\left(\frac{F_T}{1-p}\right)_i^{RK} \tag{S21}$$

$$\frac{(h_a f_{ak})_i^{m+1} - (h_a f_{ak})_i^m}{\Delta t} + (f_{Ik})_i^m \frac{\xi_i^{m+1} - \xi_i^m}{\Delta t} = -\left(\frac{F_k}{1-p}\right)_i^{RK} \tag{S22}$$

In Eqs. (S21) and (S22), the superscript RK indicates that the sediment exchange is estimated by the second-order Runge-Kutta method for the source terms. Moreover, the sediment composition in the substrate layer is also updated following the updating of the sediment composition in the active layer. In fact, the entire substrate layer is further divided into several storage layers, with the thickness of each storage layer being represented by L_s. However, the thickness of the top storage layer is variable in the range of $L \le L_s$ due to bed aggradation or degradation. In each storage layer, the sediment is assumed to be well mixed. The updating procedure of the substrate sediment composition can be classified into two cases (i.e. bed aggradation and degradation), with a detailed description of the procedure given in Qian et al. [31].

The numerical fluxes $\mathbf{G}_{i+1/2,j}$, $\mathbf{G}_{i-1/2,j}$, $\mathbf{H}_{i,j+1/2}$ and $\mathbf{H}_{i,j-1/2}$ involved in Eq. (S19a) are evaluated in the following three steps using the well-balanced surface gradient method version of the SLIC (Slope Limiter Centred) scheme. Consider the computation of $\mathbf{G}_{i+1/2,j}$. First, inter-cell variables $\mathbf{U}^L_{i+1/2,j}$ and $\mathbf{U}^R_{i+1/2,j}$ are reconstructed by extrapolating from the cell-averaged variables (i.e., $\mathbf{U}_{i,j}$) to achieve second-order accuracy in space. Second, inter-cell variables are evolved over a time step of $\Delta t/2$ to achieve second-order accuracy in time, in a similar way to Eq. (S19a). Finally, the First ORder Centred (FORCE) flux is estimated $\mathbf{G}_{i+1/2,j} = \mathbf{G}^{\text{FORCE}}(\overline{\mathbf{U}}^L_{i+1/2,j}, \overline{\mathbf{U}}^R_{i+1/2,j})$, and $\mathbf{U}^L_{i+1/2,j}$, $\mathbf{U}^R_{i+1/2,j}$ are the evolved inter-cell variables from the former two steps.

Two types of boundaries, i.e. open and closed boundaries, are involved in this work. At an open boundary, such as the inlet or outlet of a channel, the method of characteristics is used for subcritical flow conditions to obtain the updated values of flow variables, which however should be directly prescribed at the inlet and set to be zero gradients at the outlet for supercritical flows. The depth-averaged sediment concentration c_k at an open boundary, however, needs to be specified. At a closed boundary, such as the side walls of a channel, a free-slip and non-permeable condition is employed [32].

A special treatment is performed at wet-dry interfaces. If the water surface in a wet cell is lower than the bed elevation of an adjacent dry cell, then the bed elevation and water level of this dry cell are both set at the level of the water surface of the wet cell temporarily only in the flux calculation section. For example, if the cell (i, j) is wet while the adjacent cell $(i+1, j)$ is dry and $\eta_{i,j} < \eta_{i+1,j}$ then $\eta_{i,j} = z_{b\,i+1,j} = \eta_{i+1,j}$, and as a consequence the depth in the cell $(i+1, j)$ is still zero. The occurrence of very small values of water depth in numerical simulations can lead to instabilities due to the bed resistance possibly tending towards infinity, especially at wet-dry interfaces. To avoid this difficulty, the computed water depth lower than a threshold value is set to zero. Eq. (S15) for the clear-water flow layer is then solved using a similar procedure as Eq. (S16).

For numerical stability, the time step is specified according to the Courant-Friedrichs-Lewy (CFL) condition,

$$\Delta t = C_r \min\left(\frac{\Delta x}{\lambda_{wx}}, \frac{\Delta y}{\lambda_{wy}}, \frac{\Delta x}{\lambda_{mx}}, \frac{\Delta y}{\lambda_{my}}, \frac{\Delta x}{\lambda_{skx}}, \frac{\Delta y}{\lambda_{sky}}\right) \tag{S23}$$

where C_r is the Courant number. The stability limit for the SLIC scheme, which uses the FORCE method to calculate numerical fluxes, is a decreasing function of the dimension parameter χ (χ =1, 2, 3), i.e., $C_r \le \sqrt{2\chi - 1}/\chi$. For the present two-dimensional modelling, $C_r \le \sqrt{3}/2$ with χ = 2. λ_w is the eigenvalue related to motion of the clear-water flow layer; and λ_m and λ_{sk} are eigenvalues related to the motion of the water-sediment mixture and the sediment phase in the lower flow layer. In accordance, the eigenvalues are:

$$\lambda_{wx1,2} = u_w \pm \sqrt{g_{\psi_w} h_w}\,,\quad \lambda_{wy1,2} = v_w \pm \sqrt{g_{\psi_w} h_w} \tag{S24a, b}$$

$$\lambda_{m\,x1,2} = u_m \pm \sqrt{g_{\psi_m} h_m}\,,\quad \lambda_{m\,y1,2} = v_m \pm \sqrt{g_{\psi_m} h_m} \tag{S24c, d}$$

$$\lambda_{skx\,1,2} = u_{sk} \pm \sqrt{0.5(\rho_m/\rho_s) g_{\psi_m} h_m}\,,\quad \lambda_{sky\,1,2} = v_{sk} \pm \sqrt{0.5(\rho_m/\rho_s) g_{\psi_m} h_m} \tag{S24e, f}$$

Table S1 Summary of numerical cases of barrier lake formation due to sustained inflow of granular landslide and the results (Series 3)

Case	Landslide inflow discharge q_s (m^3/s)	Medium grain size d_m (m)	Initial flow velocity (m/s)	Initial flow depth h_{w0} (m)	Barrier lake	
					Formation	Time (s)
3-1	4.5	0.2	0	10	Y	33.2
3-2	4.5	0.2	1	10	Y	34.8
3-3	4.5	0.2	5	10	Y	33.9
3-4	3.0	0.2	0	10	Y	45.68
3-5	1.5	0.2	0	10	Y	58.25
3-6	0.5	0.2	0	10	N	n/a
3-7	4.5	0.01	0	10	N	n/a
3-8	4.5	0.05	0	10	N	n/a
3-9	4.5	0.1	0	10	Y	44.6
3-10	4.5	0.4	0	10	Y	28.6
3-11	4.5	0.2	0	5	Y	24.2
3-12	4.5	0.2	0	15	Y	36.5
3-13	4.5	0.2	0	25	N	n/a
3-14	4.5	0.2	2	10	Y	35.2
3-15	3.0	0.2	2	10	N	n/a
3-16	1.5	0.2	2	10	N	n/a
3-17	0.5	0.2	2	10	N	n/a
3-18	4.5	0.01	2	10	N	n/a
3-19	4.5	0.05	2	10	N	n/a
3-20	4.5	0.1	2	10	N	n/a
3-21	4.5	0.4	2	10	Y	32.6

3-22	4.5	0.2	2	5	Y	29.1
3-23	4.5	0.2	2	15	N	n/a
3-24	4.5	0.2	2	25	N	n/a

Table S2 Summary of numerical cases of barrier lake formation due to sudden failure of landslide and the results (Series 4)

Case	Valley Type	Upstream river flow discharge (m^3/s)	Bed inclination slope θ (°)	Initial landslide velocity (m/s)	Initial landslide volume (m^3/s)	Medium grain size (m)	Barrier lake	
							Formation	Time (s)
4-1	A	0.3	0	1	0.6	0.02	Y	4.06
4-2	A	0.6	0	1	0.6	0.02	N	n/a
4-3	A	1.2	0	1	0.6	0.02	N	n/a
4-4	B	0.3	0	1	0.6	0.02	Y	3.25
4-5	B	0.6	0	1	0.6	0.02	N	n/a
4-6	B	1.2	0	1	0.6	0.02	N	n/a
4-7	C	0.3	0	1	0.6	0.02	Y	2.55
4-8	C	0.6	0	1	0.6	0.02	Y	n/a
4-9	C	1.2	0	1	0.6	0.02	N	n/a
4-10	C	0.3	5	1	0.6	0.02	Y	2.21
4-11	C	0.6	5	1	0.6	0.02	N	n/a
4-12	C	1.2	5	1	0.6	0.02	N	n/a
4-13	C	0.3	0	3	0.6	0.02	Y	2.03
4-14	C	0.6	0	3	0.6	0.02	Y	4.28
4-15	C	1.2	0	3	0.6	0.02	N	n/a
4-16	C	0.3	0	1	0.1	0.02	N	n/a
4-17	C	0.3	0	1	0.2	0.02	N	n/a
4-18	C	0.3	0	1	0.4	0.02	N	n/a
4-19	C	1.2	0	1	0.1	0.02	N	n/a
4-20	C	1.2	0	1	0.2	0.02	N	n/a
4-21	C	1.2	0	1	0.4	0.02	N	n/a
4-22	C	0.3	0	1	0.6	0.005	N	n/a

4-23	C	0.3	0	1	0.6	0.01	N	n/a
4-24	C	0.3	0	1	0.6	0.04	Y	2.36
4-25	C	0.3	0	1	0.6	0.08	Y	2.02
4-26	C	1.2	0	1	0.6	0.005	N	n/a
4-27	C	1.2	0	1	0.6	0.01	N	n/a
4-28	C	1.2	0	1	0.6	0.04	N	n/a
4-29	C	1.2	0	1	0.6	0.08	Y	6.71

Table S3 Summary of computed critical indexes for barrier lake formation for Series 3 and 4

Case	Volume ratio R_V	Velocity ratio R_u	Density ratio R_ρ	Shields Number θ	Critical index I_{MP}	Barrier lake	
						Formation	Time (s)
3-1	0.540	2.000	1.990	0.870	2.470	Y	33.2
3-2	0.540	1.350	1.990	0.870	1.667	Y	34.8
3-3	0.540	0.270	1.990	0.866	0.335	Y	33.9
3-4	0.360	2.780	1.990	0.870	2.289	Y	45.68
3-5	0.180	1.875	1.990	0.870	0.772	Y	58.25
3-6	0.060	0.833	1.990	0.870	0.114	N	n/a
3-7	0.540	3.913	1.990	17.401	0.242	N	n/a
3-8	0.540	3.214	1.990	3.480	0.992	N	n/a
3-9	0.540	2.647	1.990	1.740	1.635	Y	44.6
3-10	0.540	3.051	1.990	0.435	7.537	Y	28.6
3-11	2.160	2.123	1.990	0.870	10.489	Y	24.2
3-12	0.240	2.169	1.990	0.870	1.191	Y	36.5
3-13	0.086	2.222	1.990	0.870	0.437	N	n/a
3-14	0.540	0.675	1.990	0.870	0.834	Y	35.2
3-15	0.360	0.450	1.990	0.870	0.371	N	n/a
3-16	0.180	0.225	1.990	0.870	0.093	N	n/a
3-17	0.060	0.075	1.990	0.870	0.010	N	n/a
3-18	0.540	0.675	1.990	17.401	0.042	N	n/a
3-19	0.540	0.675	1.990	3.480	0.208	N	n/a
3-20	0.540	0.675	1.990	1.740	0.417	N	n/a
3-21	0.540	0.675	1.990	0.435	1.667	Y	32.6
3-22	2.160	0.675	1.990	0.870	3.335	Y	29.1
3-23	0.240	0.675	1.990	0.870	0.371	N	n/a

3-24	0.086	0.675	1.990	0.870	0.133	N	n/a
4-1	1.667	1.700	1.990	5.229	1.078	Y	4.06
4-2	1.667	0.850	1.990	5.229	0.539	N	n/a
4-3	1.667	0.425	1.990	5.229	0.270	N	n/a
4-4	1.667	1.700	1.990	5.229	1.078	Y	3.25
4-5	1.667	0.850	1.990	5.229	0.539	N	n/a
4-6	1.667	0.425	1.990	5.229	0.270	N	n/a
4-7	1.667	1.700	1.990	5.229	1.078	Y	2.55
4-8	1.667	0.850	1.990	5.229	0.539	Y	n/a
4-9	1.667	0.425	1.990	5.229	0.270	N	n/a
4-10	1.667	1.700	1.990	5.229	1.078	Y	2.21
4-11	1.667	0.850	1.990	5.229	0.539	N	n/a
4-12	1.667	0.425	1.990	5.229	0.270	N	n/a
4-13	1.667	5.100	1.990	5.307	3.188	Y	2.03
4-14	1.667	2.550	1.990	5.307	1.594	Y	4.28
4-15	1.667	1.275	1.990	5.307	0.797	N	n/a
4-16	0.278	1.700	1.990	5.229	0.180	N	n/a
4-17	0.556	1.700	1.990	5.229	0.360	N	n/a
4-18	1.111	1.700	1.990	5.229	0.719	N	n/a
4-19	0.278	0.425	1.990	5.229	0.045	N	n/a
4-20	0.556	0.425	1.990	5.229	0.090	N	n/a
4-21	1.111	0.425	1.990	5.229	0.180	N	n/a
4-22	1.667	1.700	1.990	20.916	0.270	N	n/a
4-23	1.667	1.700	1.990	10.458	0.539	N	n/a
4-24	1.667	1.700	1.990	2.614	2.157	Y	2.36
4-25	1.667	1.700	1.990	1.307	4.315	Y	2.02
4-26	1.667	0.425	1.990	20.916	0.067	N	n/a
4-27	1.667	0.425	1.990	10.458	0.135	N	n/a

4-28	1.667	0.425	1.990	2.614	0.539	N	n/a
4-29	1.667	0.425	1.990	1.307	1.079	Y	6.71

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