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# Spectroscopy of chimera baryons in a $Sp(4)$ lattice gauge theory

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Chimera baryons are an important element of strongly coupled theories that provide a microscopic origin for UV complete composite Higgs models (CHMs), since they play the role of top partners in top partial compositeness. In a particular interesting realisation of CHMs based upon an underlying  $Sp(4)$  gauge theory, such exotic objects are composed of two fermion constituents transforming on the fundamental, and one on the 2-index antisymmetric representations. We perform lattice computations of the chimera baryon spectrum in the quenched approximation. We present preliminary results for the masses of various chimera baryons with different quantum numbers, including the one interpreted as the top partner. We test the technology needed for future calculations with dynamical fermions.

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## 1. Introduction

Our interest in the study of  $Sp(4)$  gauge theories arises from the contexts of CHMs [1–3]. References [4, 5] show that the  $Sp(4)$  gauge theory with two fermionic fields in fundamental and three in 2-index antisymmetric representations is a CHM candidate with  $SU(4)/Sp(4)$  coset. It is the minimal model containing 5 pseudo Nambu–Goldstone bosons; 4 of them can be interpreted as the Higgs doublet in the Standard Model (SM) and 1 can be made heavy. An exotic object, called chimera baryon, can mix with the standard model top quark, providing mass to the latter through partial compositeness. This mechanism for generating the top mass was firstly introduced in Ref. [6]. The first lattice study of chimera baryons was presented in Ref. [7] for a  $SU(4)$  gauge theory.

Our collaboration has been conducting studies of  $Sp(2N)$  gauge theories by employing well-developed lattice technology [8–16] (see Refs. [14, 15] for the discussion and preliminary results for chimera baryons). We aim to launch the mass spectrum calculations of  $Sp(4)$  gauge theory with dynamical Dirac fermions in two distinct representations. As a preliminary step before doing highly demanding dynamical lattice simulations, we study the spectrum of chimera baryons with quenched fermions to guide us on bare lattice parameter space when generating gauge configurations. In this article, we present the interpolating operator of chimera baryons and our preliminary results in the quenched approximation.

## 2. Interpolating operators and measurements

In order to build a spin-1/2 composite fermion neutral under  $Sp(4)$  gauge group, the simplest way is to combine two fundamental fermions (denoted by  $Q$ ) and one 2-index antisymmetric fermion (denoted by  $\Psi$ ). The interpolating operator reads

$$O_{\text{CB}}^\gamma(x) \equiv \left( Q^{i a}_\alpha(x) \Gamma^{1 \alpha \beta} Q^{j b}_\beta(x) \right) \Omega_{ad} \Omega_{bc} \Gamma^{2 \delta \gamma} \Psi^{k cd}_\gamma(x), \quad (1)$$

where  $a, b, c, d$  are colour indices,  $i, j, k$  are flavor indices,  $\alpha, \beta, \delta, \gamma$  are spinor indices,  $\Gamma^{1,2}$  are combinations of gamma matrices and  $\Omega$  is the  $4 \times 4$  symplectic matrix. The  $SU(3)_c$  SM gauge group is a subgroup of the flavour symmetry acting on  $\Psi^k$ . The Dirac conjugate of the interpolating operator is

$$\overline{O_{\text{CB}}^\gamma}(x) \equiv \overline{\Psi^{k cd}_\delta(x)} \Omega_{cb} \Omega_{da} \Gamma^{2 \delta \gamma} \left( \overline{Q^{j b}_\beta(x)} \Gamma^{1 \beta \alpha} \overline{Q^{i a}_\alpha(x)} \right). \quad (2)$$

Considering the case  $i \neq j$ , the zero momentum two-point correlation function is

$$\begin{aligned} C^{\gamma \gamma'}(t) &\equiv \sum_{\vec{x}} \langle O_{\text{CB}}^\gamma(x) \overline{O_{\text{CB}}^{\gamma'}}(0) \rangle \\ &= - \sum_{\vec{x}} \left( \Gamma^2 S_\Psi^{k cd}_{c'd'}(x, 0) \overline{\Gamma^2} \right)_{\gamma \gamma'} \Omega_{cb} \Omega^{b'c'} \Omega_{ad} \Omega^{d'a'} \\ &\quad \times \text{Tr} \left[ \Gamma^1 S_Q^b_{b'}(x, 0) \overline{\Gamma^1} S_Q^a_{a'}(x, 0) \right], \end{aligned} \quad (3)$$

where we define  $x \equiv (t, \vec{x})$  and  $\overline{\Gamma} \equiv \gamma^0 \Gamma^\dagger \gamma^0$ , the trace is over the spinor indices, and the fermion propagators are defined as

$$S_Q^{i a}_{b \alpha \beta}(x, y) = \langle Q^{i a}_\alpha(x) \overline{Q^{i b}_\beta}(y) \rangle \text{ and } S_\Psi^{k ab}_{cd \alpha \beta}(x, y) = \langle \Psi^{k ab}_\alpha(x) \overline{\Psi^{k cd}_\beta}(y) \rangle. \quad (4)$$

The interpolating operator of top partner chimera baryon has been written down in Ref. [15]. One uses  $(\Gamma^1, \Gamma^2) \equiv (C\gamma^5, \mathbb{1})$ . Other choices of gamma matrices can be found in Ref. [17] for the  $SU(4)$  gauge theory. In analogy with QCD, we denote as  $\Lambda$  the state with quantum numbers  $(J, R) = (1/2, 5)$ , where  $J$  is the spin and  $R$  denotes the irreducible representation of the flavour group acting on  $Q^i$ . This state provides a natural candidate as a top partner [18]. At large Euclidean time, the 2-point correlation function for this  $\Lambda$ -type operator asymptotically behaves as

$$C(t) \xrightarrow{t \rightarrow \infty} P_e \left[ c_e e^{-m_e t} + c_o e^{-m_o(T-t)} \right] - P_o \left[ c_o e^{-m_o t} + c_e e^{-m_e(T-t)} \right], \quad (5)$$

where  $P_e$  and  $P_o$  are parity projectors defined as

$$P_e \equiv \frac{1}{2}(1 + \gamma^0) \text{ and } P_o \equiv \frac{1}{2}(1 - \gamma^0). \quad (6)$$

We denote as  $c_e$  and  $c_o$  the coefficients,  $m_e$  and  $m_o$  the mass of parity even and odd states, respectively.  $T$  is the temporal lattice size. As we see from Eq. (5), the backward propagator of the even projected correlator,  $C_e(t) \equiv P_e C(t)$ , decays with the same mass as the forward propagator of the odd projected correlator,  $C_o(t) \equiv P_o C(t)$ , and vice versa. Thus, we make use of two projections to improve the statistics by averaging with the function

$$\tilde{C}(t) = \frac{1}{2} [C_e(t) + C_o(T-t)] \xrightarrow{t \rightarrow \infty} \tilde{c}_e e^{-m_e t} + \tilde{c}_o e^{-m_o(T-t)}. \quad (7)$$

We also measure other types of chimera baryons, for completeness. Other possible low lying states are  $\Sigma$ :  $(J, R) = (1/2, 10)$  and  $\Sigma^*$ :  $(J, R) = (3/2, 10)$ , which share the same interpolating operator constructed using  $(\Gamma^1, \Gamma^2) \equiv (C\gamma^\mu, \mathbb{1})$  in Eq. (1). In general, the  $\Sigma$  state could also play the role of the top partner [19]. The two-point function is defined as

$$C_{ij}^{\gamma\gamma'}(t) \equiv \sum_{\vec{x}} \langle O_{CB,i}^\gamma(x) \overline{O_{CB,j}^{\gamma'}}(0) \rangle, \quad (8)$$

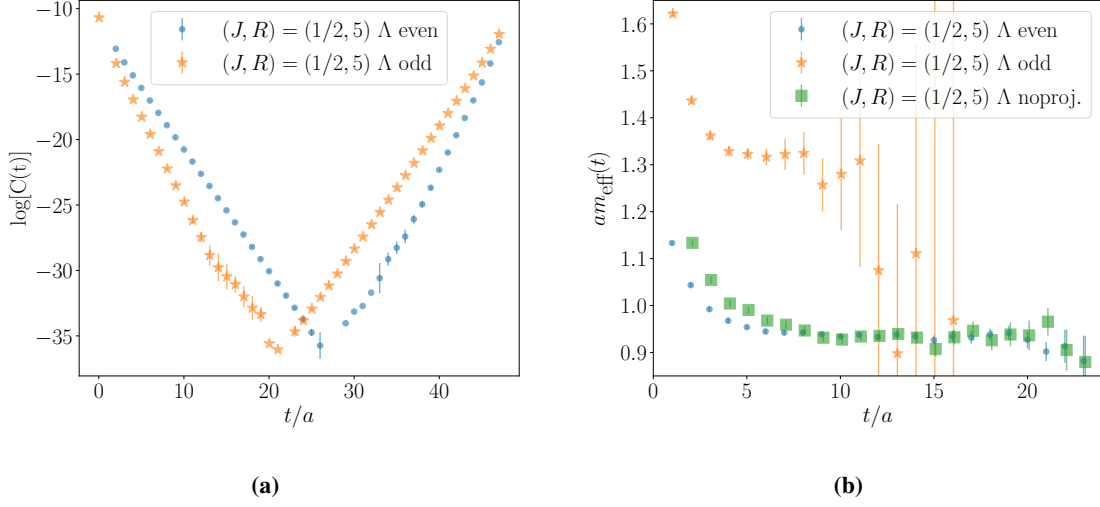
where  $i$  ( $j$ ) refers to the case of  $\Gamma^1 = C\gamma^i$  with  $i = 1, 2, 3$ . The spin-1/2 and -3/2 states are isolated by employing projectors (see Ref. [20] for the analogous treatment of heavy baryon in QCD):

$$P_{ij}^{1/2} \equiv \frac{1}{3}\gamma_i\gamma_j \text{ and } P_{ij}^{3/2} \equiv \delta_{ij} - \frac{1}{3}\gamma_i\gamma_j, \quad (9)$$

for spin-1/2 states  $C_{1/2}^{\gamma\gamma'}(t) \equiv P_{ij}^{1/2} C_{ji}^{\gamma\gamma'}(t)$ , and for spin-3/2 states  $C_{3/2}^{\gamma\gamma'}(t) \equiv P_{ij}^{3/2} C_{ji}^{\gamma\gamma'}(t)$ .

### 3. Mass spectrum

Ensembles are generated using the same setup of an earlier study of the mesonic spectroscopy with quenched fermions [11]. We aim at exploring how chimera baryon masses depend on the fermion masses. To improve the signal, we employ Wuppertal (Gaussian) smearing [21] and APE smearing [22] to construct operators with better overlap with the ground state.



**Figure 1:** (a) Log plot of correlators and (b) effective mass plot comparison with parity projections in units of lattice spacing  $a$ . The blue circles and orange stars denote the even and odd projected correlators, respectively. The green squares in the right panel denote the original correlators without projection. The calculations are performed on a  $48 \times 24^3$  lattice (QB1 in Ref. [11]).

### 3.1 Parity and spin projections

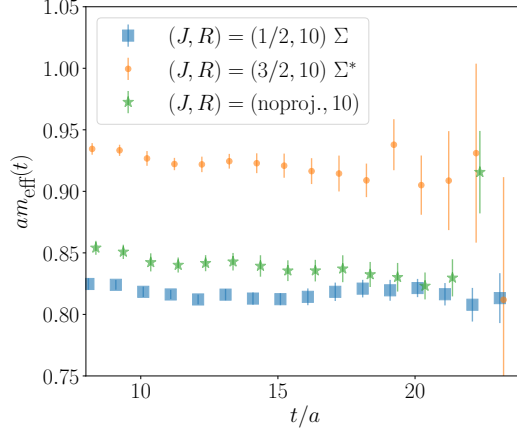
We demonstrate the conjugate propagation properties of even and odd states in Fig. 1a. The correlators of two parity states behaves as Eq. (5). For instance, the forward propagating state with even projection is the same as the backward propagating state with odd projection. Because of such an asymmetry in the correlation function, we do not use the usual cosh function for the extraction of masses. In principle, we could extract the masses for both parity even and odd states by fitting data to Eq. (7). However, for this preliminary study we focus on the lightest one, and the masses are determined by performing a single exponential fit to  $\tilde{C}(t)$  over the time range of the plateau in the effective mass defined as

$$m_{\text{eff}}(t) \equiv \ln \left( \frac{\tilde{C}(t)}{\tilde{C}(t+1)} \right). \quad (10)$$

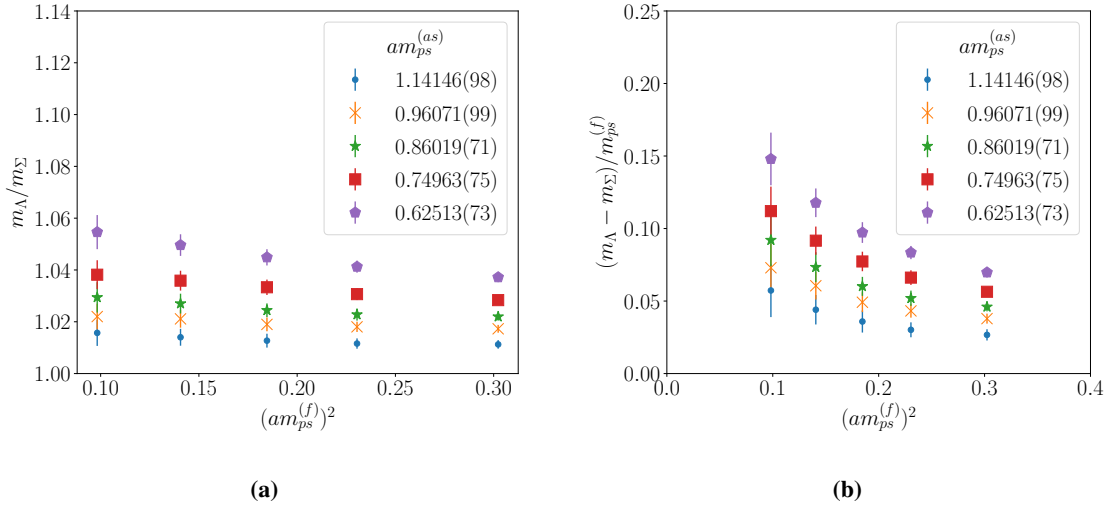
As shown in Fig. 1b, we find that the mass of the parity-even state is statistically the same as the mass without projection at large Euclidean time. Since the mass of the parity-even state is lower than that of the parity-odd state and measured with a higher precision, we focus on the former. We further find that parity projection significantly reduces statistical errors of the parity-even masses. Similar phenomena can be found with the spin projections, see Fig. 2. The effective mass at large Euclidean time without spin projection is compatible with that of the spin-1/2 state but has larger error, and the spin-3/2 state is heavier than the spin-1/2 state.

### 3.2 Mass hierarchy of chimera baryons and fermion mass dependence

In order to study how the constituent fermion masses affect the chimera baryon masses, we examine their dependence on the pseudoscalar meson mass. We extract the chimera baryon masses by fitting the correlators with a single exponential function, while using the cosh function for the analysis of meson correlators. As a first step, we vary the bare masses on one single ensemble (the



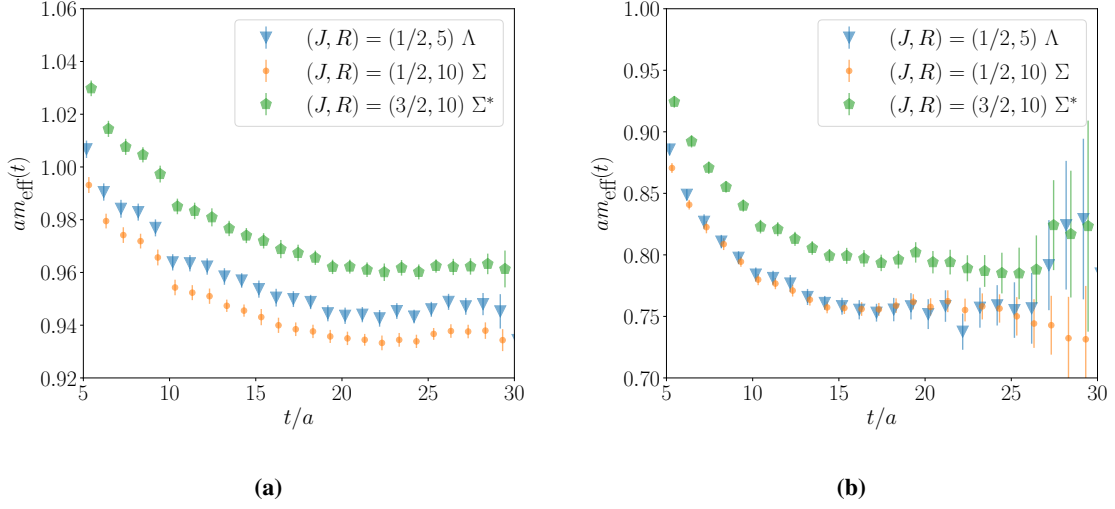
**Figure 2:** Comparison of effective mass plots between two spin states and the state without spin projection. At large Euclidean time, the spin-1/2 state (blue squares), which is compatible with the mass obtained without spin projection (green stars), is lighter than spin-3/2 state (orange points).



**Figure 3:** Plot of (a) the mass ratio  $m_{\Lambda}/m_{\Sigma}$  and (b) the ratio  $(m_{\Lambda} - m_{\Sigma})/m_{ps}^{(f)}$  against fundamental pseudoscalar meson mass square  $(am_{ps}^{(f)})^2$ . Different colours and shapes represent measurements at different antisymmetric pseudoscalar meson masses.

smallest lattice volume with largest lattice spacing, QB1 in Ref. [11]). One interesting feature we immediately discovered is that the  $\Lambda$  chimera baryon is not the lightest state; as demonstrated by Fig. 3a. Under the assumption that  $\Lambda$  is the top partner, it should be a stable state, for model-building purposes. We examine the naive possible decay process of  $\Lambda$  into  $\Sigma$  and a pseudoscalar meson by checking the ratio  $(m_{\Lambda} - m_{\Sigma})/m_{ps}^{(f)}$ . As long as the ratio is less than one, the  $\Lambda$  chimera baryon would be stable. As shown in Fig 3b, this is the case for all our current simulations.

Figure 3a also shows that  $m_{\Lambda}/m_{\Sigma}$  becomes smaller when the fundamental pseudoscalar meson mass increases. Similar effects are observed when increasing the antisymmetric pseudoscalar



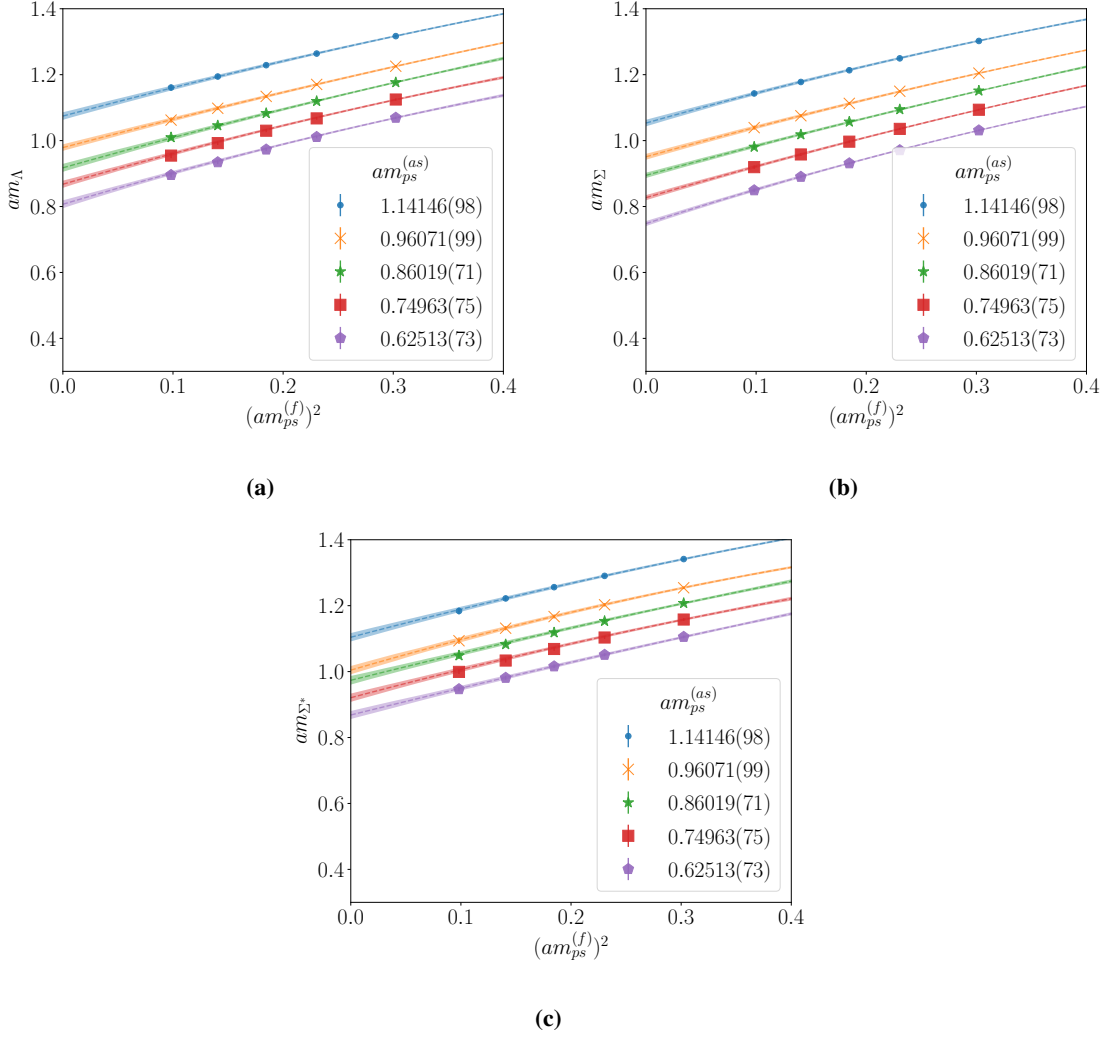
**Figure 4:** Effective mass plot of chimera baryons calculated with different fundamental bare masses **(a)**  $m_0^{(f)} = -0.6$  and **(b)**  $m_0^{(f)} = -0.69$ , at fixed antisymmetric bare mass  $m_0^{(as)} = -0.81$ , on a  $60 \times 48^3$  finer lattice (QB4 in Ref. [11]).

meson mass. It should be noted that the heaviest pseudoscalar meson mass in the antisymmetric representation is already above the cut-off scale. We further illustrate such effects on our finest lattice with a volume,  $60 \times 48^3$  (QB4 in Ref. [7]). Figure 4 exhibits that the  $\Sigma$  baryon is clearly lighter than the  $\Lambda$  baryon at  $m_{ps}^{(as)}/m_{ps}^{(f)} \sim 1.8$ . On the other hand, if we decrease the fundamental bare mass while holding fixed the antisymmetric one, the masses of  $\Lambda$  and  $\Sigma$  become compatible with one another when  $m_{ps}^{(as)}/m_{ps}^{(f)} \sim 4.46$ . This suggests that at sufficient large antisymmetric pseudoscalar meson mass and relative small fundamental pseudoscalar meson mass, there is possibility that the  $\Lambda$  chimera baryon is the lightest state. We are currently completing the investigation on all the ensembles. Our study of the relation between pseudoscalar meson masses and baryon mass hierarchy has implications for constructing viable CHMs with partial compositeness.

We now turn our attention to the mass dependence of an individual chimera baryon. We perform an extrapolation towards the limit of massless fundamental pseudoscalar meson. As a first attempt with one single ensemble (QB1), the extrapolation of the chimera baryon mass is performed at fixed antisymmetric pseudoscalar meson mass with a naive polynomial function including  $(am_{ps}^{(f)})^2$  and  $(am_{ps}^{(f)})^4$  terms. See Fig. 5 for the results. In future work, we will make this exploratory study more systematic and rigorous by using baryonic chiral perturbation theory [23].

#### 4. Conclusion and outlook

In the  $Sp(4)$  gauge theory with two fundamental and three antisymmetric Dirac fermions, a candidate to UV-complete CHMs with partial compositeness, chimera baryons play an important role as the top partners. In this contribution, we have presented the interpolating operators for chimera baryons, and the analysis of relevant correlators for extracting their masses. We have reported preliminary results on the masses of three chimera baryon states,  $\Lambda$ :  $(J, R) = (1/2, 5)$ ,  $\Sigma$ :  $(J, R) = (1/2, 10)$  and  $\Sigma^*$ :  $(J, R) = (3/2, 10)$ . One important result for model building we



**Figure 5:** Mass extrapolation of (a)  $\Lambda$ , (b)  $\Sigma$  and (c)  $\Sigma^*$  chimera baryons towards massless fundamental pseudoscalar meson at fixed antisymmetric pseudoscalar meson masses (different colours and shapes), which are performed by fitting a polynomial function with respect to  $(am_{ps}^{(f)})^2$ .

found is that the mass hierarchy of chimera baryons depends on the pseudoscalar meson masses. These are preliminary results, mostly measured on one quenched ensemble. We are currently performing analysis on all the quenched ensembles available to us, that will serve as a guide to future investigations with fully dynamical simulations.

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