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Bending Stresses of Steel Web Tapered Tee Section Cantilevers

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Abstract: Although commonly used, no design method is available for steel web tapered tee section cantilevers. This paper investigates the bending stresses of such beams. Relationships between the maximum compressive stress and the degree of taper were investigated. An analytical model is presented to determine the location of the maximum stress when subjected to a uniformly distributed load or a point load at the free end and was validated using finite element analysis and physical tests. It was found that the maximum stress always occurs at the support when subjected to a uniformly distributed load. When subjected to a point load at the free end and the degree of taper is up to seven, it was found that Miller's equation could be used to determine the location of the maximum stress. However, it is shown that when the degree of taper is greater than seven, Miller's equation does not accurately predict the location and the analytical model should be used. It was also found that the location of the maximum stress was solely dependent on the degree of taper, while a geometric ratio, β was required to determine the magnitude of the maximum stress. A simple method that predicts the magnitude of the maximum stress is proposed. The average error in the prediction of the magnitude of the maximum stress is found to be less than 1.0%.

Key words: Web tapered tee sections, tapered cantilevers, bending stress patterns, maximum stress shift, finite element analysis.

1. Introduction

Steel web tapered cantilevers are used because of their aesthetic features and lightweight. They are structurally efficient because the web is tapered along the beam to closely match the variation of the bending moment of the beam. The depth of the beam is largest at the fixed support, where its bending moment is greatest and gradually decreases towards the free end.

Although steel web tapered tee cantilevers are commonly used, neither Eurocode 3 nor BS 5950 provides a design method for such beams. In BS 5950 [1], some design rules exist for untapered tees and tapered beams whose cross-sections are other than tee sections. The majority of the existing literature deals with the lateral torsional buckling capacities of tapered I-beams [2-6] or tapered channels [3]. Tapered tee cantilevers were studied by Fischer and Smida [7] and Yuan et al. [8]. However, their study dealt with the instabilities of the beams.

The design of tapered rectangular levers with a point load at the tip was studied by Miller [9]. Miller pointed out that the location of the maximum bending stress of the tapered levers moved away from the support as the degree of taper was more than two and determined the location of the maximum stress as follows:

$$\text{for} \quad \frac{h}{h'} > 2, \quad x' = \frac{L}{\frac{h}{h'} - 1} \tag{1}$$

where, h and h' are the depths of the beam at the support and the tip, respectively, and x' is the distance from the tip and L is the cantilever length.

Eq. (1) can be rearranged using the distance from the support, x ($x = L - x'$) as follows:

$$\text{for} \quad \frac{h}{h'} > 2, \quad \frac{x}{L} = \frac{\frac{h}{h'} - 2}{\frac{h}{h'} - 1} \tag{2}$$

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However, Miller's study was limited to rectangular cross-sections and the stress patterns for other cross-sections may differ. In a study with tapered I-beams [2], the maximum stress, hence the plastic hinge, of the tapered I-beam cantilevers occurred away from the support when the degree of taper was high. However, the location of the maximum stress was not quantified.

This paper investigates the bending stresses of steel web tapered tee section cantilevers. Cantilevers with various degree of taper were analyzed. The stress pattern along the beam is identified and the location and magnitude of the maximum bending stress are discussed and simple methods are suggested to predict the latter two.

2. Web Tapered Tee Cantilevers

In the absence of design guidance, a conventional, yet onerous, analysis may be used for the design of web tapered tee cantilevers. The resistance and stiffness of such beams may then be checked along the tapered beam using the section properties at each point [10]. The bending stress of a tapered beam may be calculated using the following well known equation:

$$\sigma = \frac{My}{I} \tag{3}$$

where, σ is the bending stress on any fibre at a distance y from the neutral axis, M is the bending moment and I is the second moment of area about the neutral axis.

2.1 Bending Stresses

As the cross-section of a tee is mono-symmetric and the neutral axis lies towards the flange, the compressive stress is more critical than the tensile stress. Fig. 1 shows the theoretical (using Eq. (3)) compressive stress along the beam relative to that at the support. A 152 kg/m × 229 kg/m × 34 kg/m tee section cut from a 457 × 152 × 67 UKB was used. The dimensions of the cantilever beams are given in Table 1. Five cases distinguished by the degree of taper (*h*/*h'*) were analyzed. The degree of taper, *h*/*h'* is defined by the ratio of the depth at the support (*h*) to that at the tip (*h'*), as shown in Fig. 2.

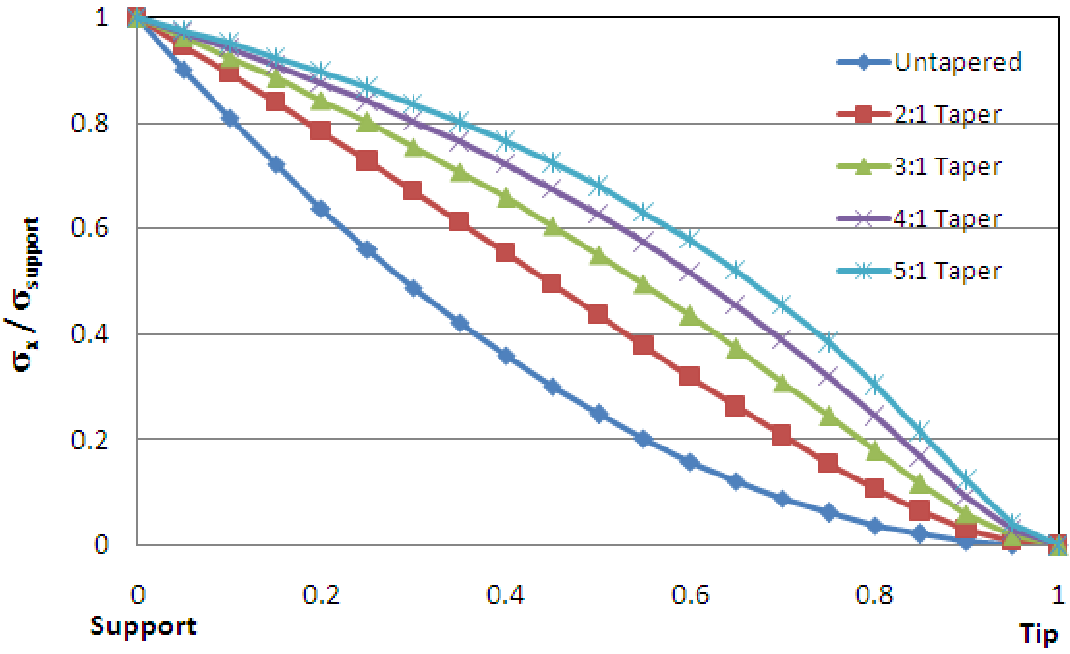
When subjected to a uniformly distributed load the maximum compressive stress always occurs at the support regardless of the degree of taper, as shown in Fig. 1a. While the location of the maximum stress changes, depending on the degree of taper when subjected to a point load at the tip, as shown in Fig. 1b.

For a cantilever with a tip point load the stresses in the untapered section decrease linearly towards the tip, with the maximum at the support, due to the constant value of the section modulus. On the other hand, the stresses in the tapered sections do not decrease linearly towards the tip because of the different values of the section modulus along the beam due to the taper. It should be noted that as the degree of taper increases, the location of the maximum compressive stress moves towards the tip, as shown in Fig. 1b. This indicates that the maximum stress does not always occur at the support in tapered tees, even if the maximum bending moment occurs at the support, and therefore the plastic hinge is likely to be formed away from the support. This point was also noted in tapered I-beams [2] and tapered levers with rectangular cross-sections [9]. The maximum compressive stresses of the 3:1, 4:1 and 5:1 tapered beams are 1.1, 1.3 and 1.5 times more than that of the untapered one, even though the maximum bending moment is the same for all cases. This should be considered in the design of tapered tee cantilevers.

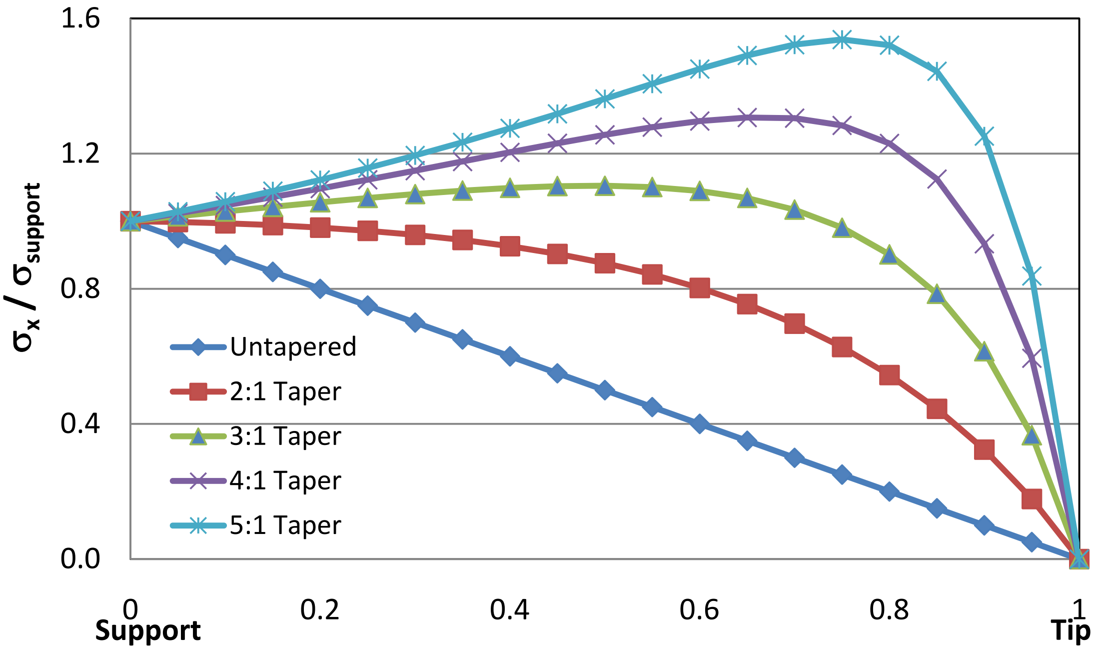
2.2 The Location of the Theoretical Maximum Bending Stress

It would be beneficial for designers to determine the location and magnitude of the maximum compressive stress of tapered cantilevers. The latter is discussed in the following section.

Consider a web tapered tee-section cantilever subject to a uniformly distributed load and a concentrated load at its free end, as shown in Fig. 3. Let x be the longitudinal axis of the beam, y and z be the cross-sectional axes parallel to the web and flange, respectively. For convenience, the origin of coordinates was chosen to be the centroid of the section. Due to the



(a) Under a uniformly distributed load



(b) Under a point load at the tip

Fig. 1 The predicted compressive stresses along the beam.

Table 1 Dimensions of the cantilevers.

Beam designation	*h*/*h'*	*h* (mm)	*h'* (mm)	Flange width (mm)	Flange thickness (mm)	Web thickness (mm)
1A and 1B	1.0	228.9	228.9	153.8	15	9
2A and 2B	2.0	228.9	114.6	153.8	15	9
3A and 3B	3.0	228.9	76.0	153.8	15	9
4	4.0	228.9	57.2	153.8	15	9
5	5.0	228.9	45.8	153.8	15	9

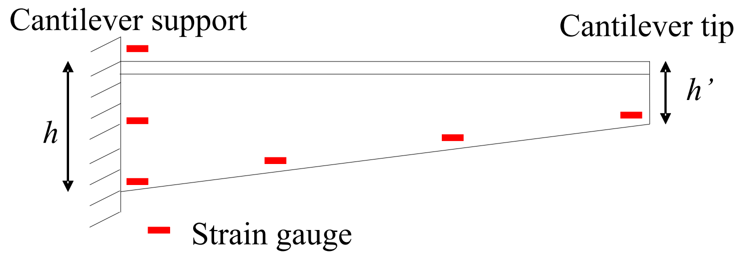


Fig. 2 Tee section cantilever showing strain gauges.

tapering of the web, the section properties of the beam are a function of the coordinate x and can be expressed as follows:

$$\bar{y} = \frac{\frac{b_f t_f^2}{2} + t_w(b_{wo} - x\tan\alpha)\left(t_f + \frac{b_{wo} - x\tan\alpha}{2}\right)}{b_f t_f + t_w(b_{wo} - x\tan\alpha)} \tag{4}$$

$$I_z = t_w(b_{wo} - x\tan\alpha)\left[\frac{(b_{wo} - x\tan\alpha)^2}{12} + \left(\frac{b_{wo} - x\tan\alpha}{2} + t_f - \bar{y}\right)^2\right] + b_f t_f\left[\frac{t_f^2}{12} + \left(\bar{y} - \frac{t_f}{2}\right)^2\right] \tag{5}$$

where, $\bar{y}$ is the distance from the top of the section to the neutral axis, I_z is the second moment of the cross-sectional area about the z-axes, b_f is the flange width, t_f is the flange thickness, b_{wo} is the web depth at the support ($x = 0$), t_w is the web thickness and α is the tapering angle.

For a beam subject to a uniformly distributed load and a concentrated load at its free end M_z can be expressed as follows:

$$M_z(x) = P(l - x) + \frac{q}{2}(l - x)^2 \tag{6}$$

where, P and q are the concentrated and uniformly distributed loads, respectively, x is the distance from the fixed support and l is the cantilever length, as shown in Fig. 3.

The compressive stress, σ_{zC} of the beam then can be determined as follows:

$$\sigma_{zC}(x) = \frac{M_z(x)}{I_z(x)}(t_f + b_w - \bar{y}) \tag{7}$$

Differentiating Eq. (7) with respect x and equating it to zero yields a value for the location of the maximum stress.

$$\frac{\partial\sigma_{zC}}{\partial x} = \frac{(t_f + b_w - \bar{y})}{I_z(x)} \cdot \frac{\partial M_z}{\partial x} - \frac{M_z(x)}{I_z^2(x)}(t_f + b_w - \bar{y})\frac{\partial I_z}{\partial x} - \frac{M_z(x)}{I_z(x)} \cdot \frac{\partial\bar{y}}{\partial x} = 0 \tag{8}$$

However, it is difficult to drive an exact formula in Eq. (8). It only gives an approximation. The exact formula for predicting the location of the maximum stress is only valid for rectangular cross-section. The locations of the maximum stress using Eq. (8) are plotted in Fig. 4 for various degrees of taper and compared with those using Miller's equation (Eq. (2)) and the theoretical stress equation, i.e., Eq. (3).

Fig. 4 shows that when subjected to a point load at the tip, the maximum compressive stress will always be at the support for a tapered tee section cantilever whose tapering ratio (h/h') is less than or equal to two. This agrees with Miller's study [9] even though the two cross-sections are different. Miller's equation can be used to determine the location of the maximum stress of a tapered tee-section cantilever whose tapering ratio is greater than two and up to seven. When the degree of taper is greater than seven, Miller's equation diverges from the rest of the two data sets, as shown in Fig. 4. When the degree of taper is greater than seven, Eq. (8) should be used to determine the location of the maximum stress.

2.3 The Magnitude of the Theoretical Maximum Bending Stress

Knowledge of the location of the theoretical maximum bending stress is very useful. However, the

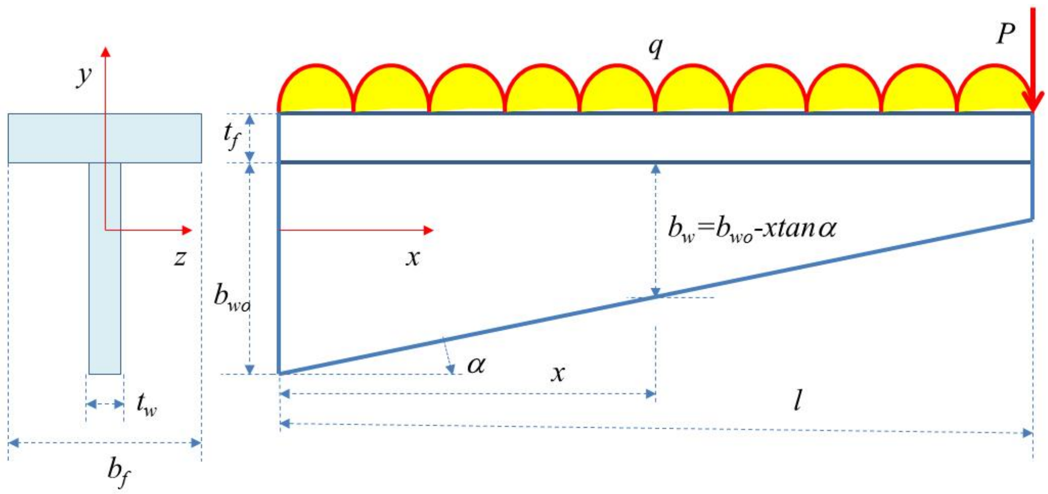


Fig. 3 A web tapered tee-section cantilever beam subject to a uniformly distributed load and a concentrated load at its free end.

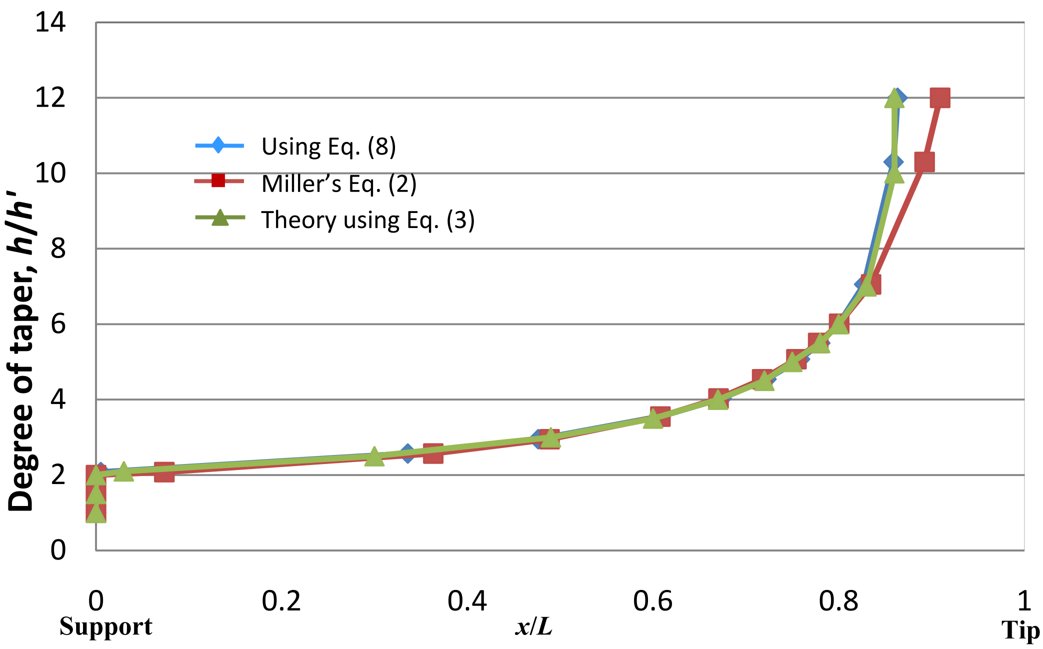


Fig. 4 Location of the maximum compressive stress under a point load at the tip.

magnitude of this stress still needs to be calculated. Achieving this requires the onerous recalculation of the location of the neutral axis and the second moment of area. Table 2 shows the relationship between the ratios of the maximum bending stress to the stress at the support, ψ and the degree of taper. 15 UK tee sections cut from UK Universal Beam sections were used. Table 2 is plotted in Fig. 5 (using a line of best fit for each tapering ratio). It was found that the stress ratio, ψ is determined by the geometric ratio, β at the support:

$$\beta = \frac{ht_w}{b_f t_f} \tag{9}$$

where, h is the height of the section and b_f is the width of the flange, and t_w and t_f are the thickness of the web and flange, respectively.

The higher value of β a section has, the lower value of ψ. The lowest and highest β values hence represent the upper and lower curves respectively in Fig. 5. A high value of β means that the web is dominant and a low value of β means that the flange is dominant. Fig. 6

shows the effect of tapering the web on the component of the stress equation. As the web is tapered, the second moment (I) of area decreases rapidly due to the d^3 term (where $I = bd^3/12$, and b and d are the breath and height of an individual section.), while the distance to the neutral axis, y_{bar} decreases less rapidly.

3. Finite Element Analyses

Prior to the experimental work a FEA (finite element analysis) was carried out to predict the stress patterns and stiffness of a web tapered tee cantilever. The same sections used in the experiments and also in Section 2.1 were analyzed. A 152 × 229 × 34 tee section cut from a 457 × 152 UKB67 was modeled in three dimensions using a finite element analysis software program, LUSAS [11]. An eight node thin shell element was used. Five cantilevers whose support to tip depth ratios of 1:1, 2:1, 3:1, 4:1 and 5:1 were analyzed. Fig. 7 shows the bending stresses along the beam when a point load of 15 kN was applied at the tip.

It should be noted that as the degree of taper increases, so does the area of compression in the web, as shown in Fig. 7. As expected, the location of the maximum stress is away from the support for the 3:1, 4:1 and 5:1 taper cantilevers. The results from the FEA are compared with the theory and the experiment in the Section of Results and Discussion.

4. Experimental Work

A total of six cantilever beams were tested in the Heavy Structures Laboratory at Plymouth University. Three sets with support to tip depth ratios of 1:1, 2:1 and 3:1 were prepared with two identical beams in each set. To simulate a cantilever, two tee beams were connected to each side of a column using extended end plates and an equal load was applied to each cantilever simultaneously, as illustrated in Fig. 8. Fig. 9 shows one side of this test arrangement. The beams were inverted, i.e., the top flange was down, so that loading could be applied from the floor using a hydraulic jack, as shown in Fig. 9.

The bending strains along the beam were measured using 6 uni-directional strain gauges. The locations of the gauges are shown in Fig. 2. Three more strain gauges were installed at the mid-span, and 100 mm before and after the mid-span of the 3:1 tapered beams in order to determine the location and the magnitude of the maximum stress. The deflections of the beams at the tip were also measured.

Table 2 Ratios of the maximum bending stress to the stress at the support (ψ).

Section	β	Degree of taper (h/h')									
(From top to bottom of Fig. 5)	(ht_w/b_ft_f)	2	2.25	2.5	2.75	3	3.5	4	4.5	5	5.5
146 × 127 × 22	0.50	1	1.012	1.042	1.082	1.128	1.231	1.339	1.447	1.552	1.651
305 × 305 × 90	0.60	1	1.009	1.036	1.074	1.119	1.220	1.331	1.446	1.561	1.675
165 × 152 × 20	0.54	1	1.009	1.036	1.074	1.118	1.219	1.329	1.444	1.560	1.675
127 × 152 × 24	0.80	1	1.009	1.035	1.072	1.117	1.218	1.329	1.444	1.559	1.672
178 × 203 × 37	0.68	1	1.009	1.035	1.072	1.116	1.217	1.328	1.443	1.558	1.673
191 × 229 × 45	0.72	1	1.008	1.034	1.071	1.115	1.215	1.325	1.440	1.556	1.672
229 × 305 × 70	0.79	1	1.007	1.032	1.068	1.110	1.209	1.318	1.433	1.549	1.667
152 × 229 × 34	0.89	1	1.006	1.029	1.064	1.105	1.201	1.308	1.421	1.538	1.656
292 × 413 × 113	0.87	1	1.006	1.029	1.063	1.105	1.201	1.307	1.420	1.536	1.655
210 × 267 × 46	0.82	1	1.006	1.029	1.063	1.104	1.200	1.306	1.419	1.535	1.653
229 × 305 × 57	0.85	1	1.006	1.029	1.063	1.104	1.197	1.306	1.419	1.535	1.653
140 × 203 × 23	0.86	1	1.005	1.028	1.062	1.102	1.196	1.301	1.413	1.529	1.646
267 × 381 × 87	0.95	1	1.005	1.027	1.060	1.100	1.193	1.297	1.408	1.523	1.641
305 × 457 × 127	0.93	1	1.005	1.028	1.061	1.102	1.196	1.302	1.414	1.529	1.648
102 × 152 × 13	1.24	1	1.003	1.022	1.052	1.088	1.175	1.272	1.377	1.486	1.599

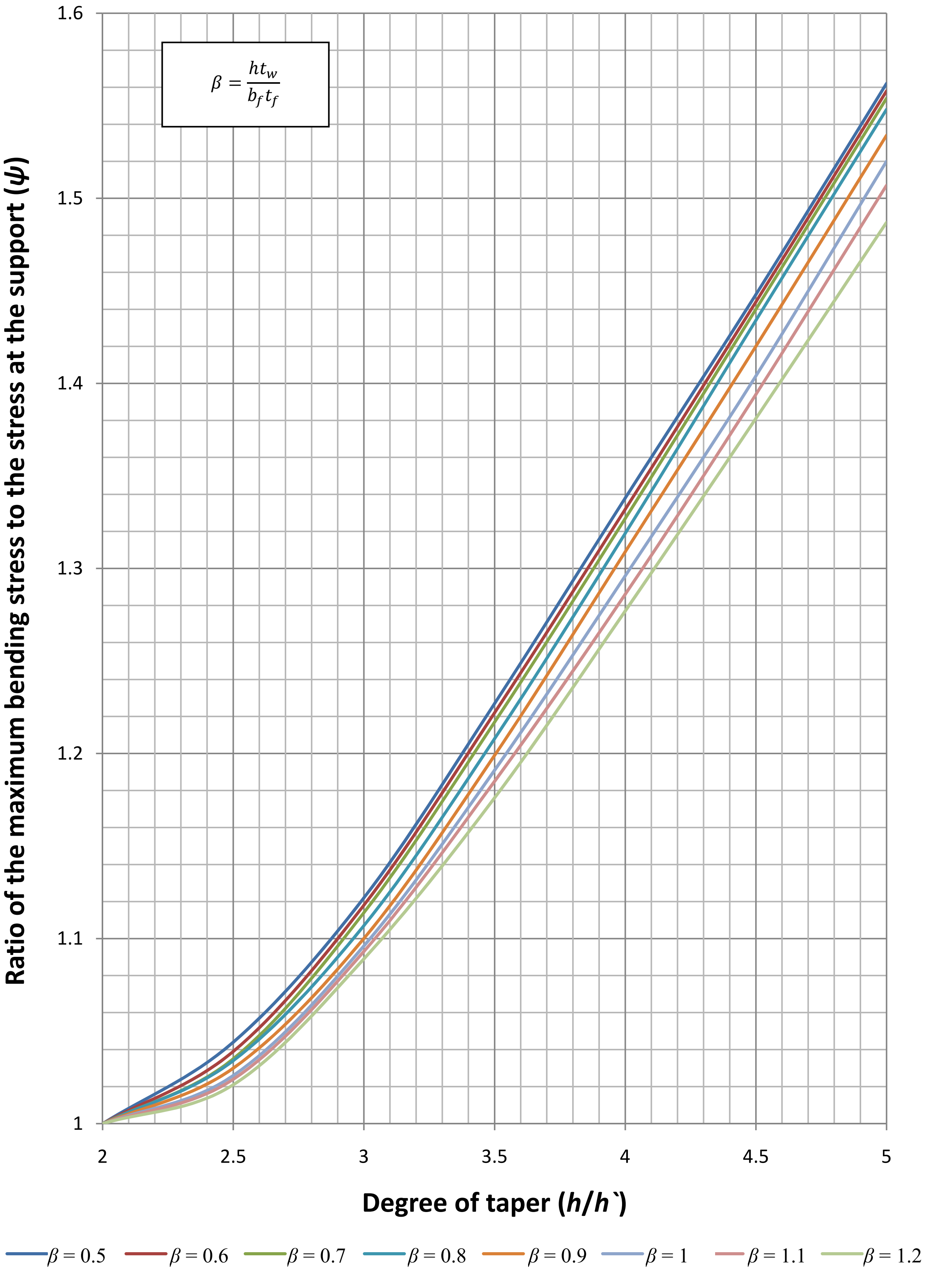


Fig. 5 Relationship between the stress ratio (ψ) and the degree of taper (h/h').

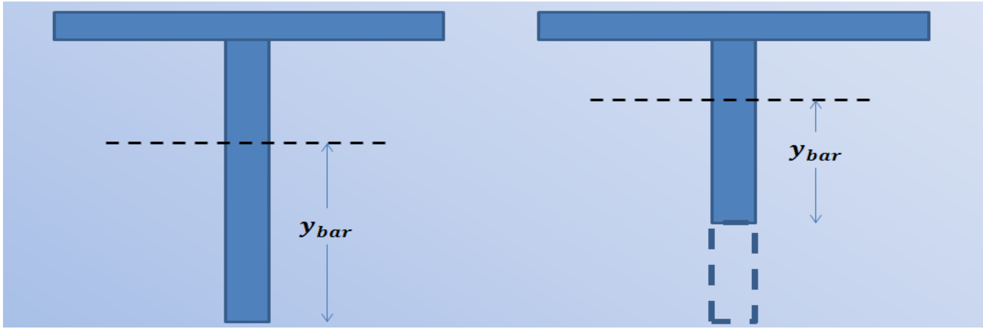
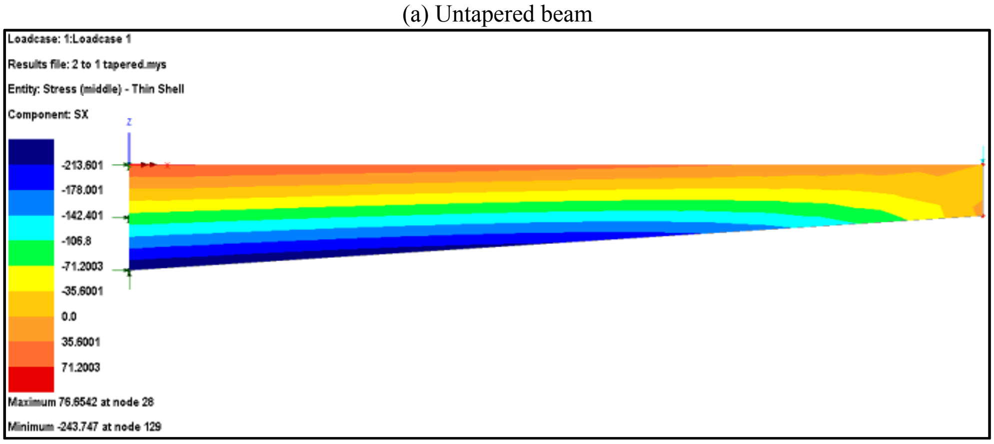
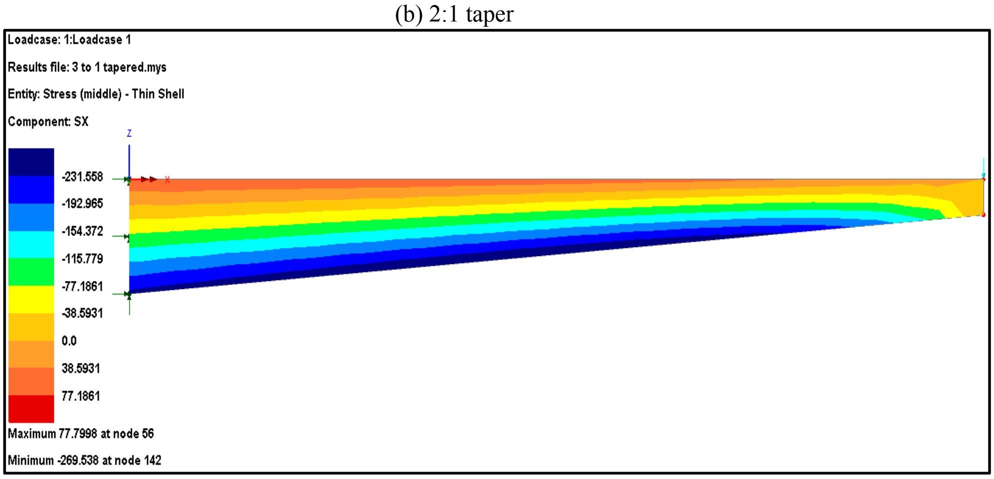


Fig. 6 The effect of web tapering on the magnitude of the stress.

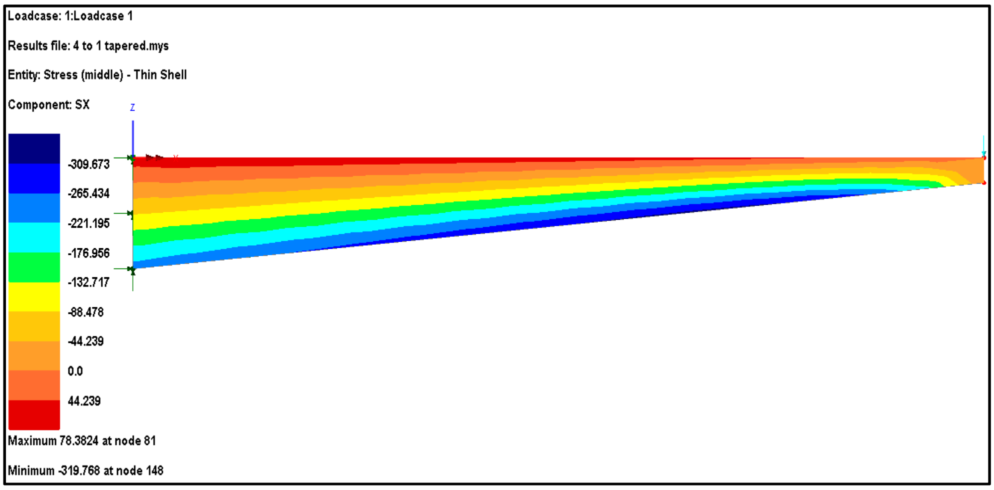
(a) Untapered beam



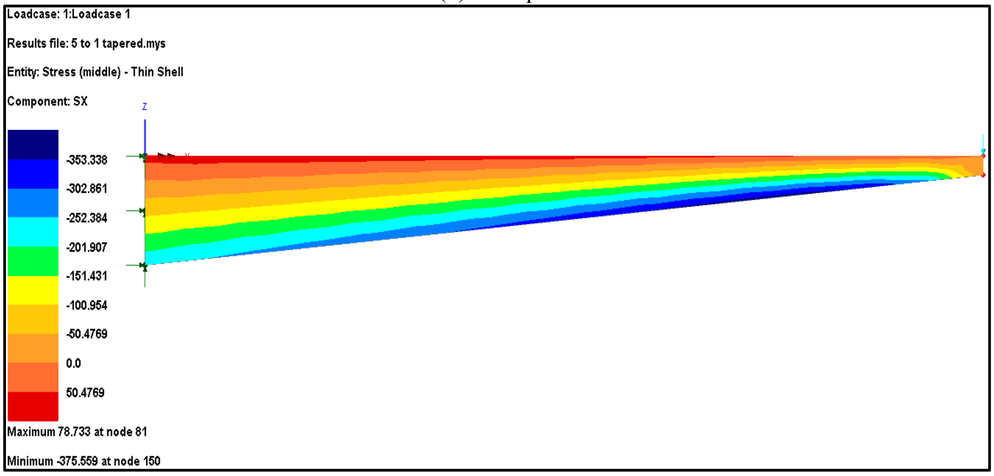
(b) 2:1 taper



(c) 3:1 taper



(d) 4:1 taper



(e) 5:1 taper

Fig. 7 The bending stresses along the beam when a point load of 15 kN was applied at the tip (N/mm^2).

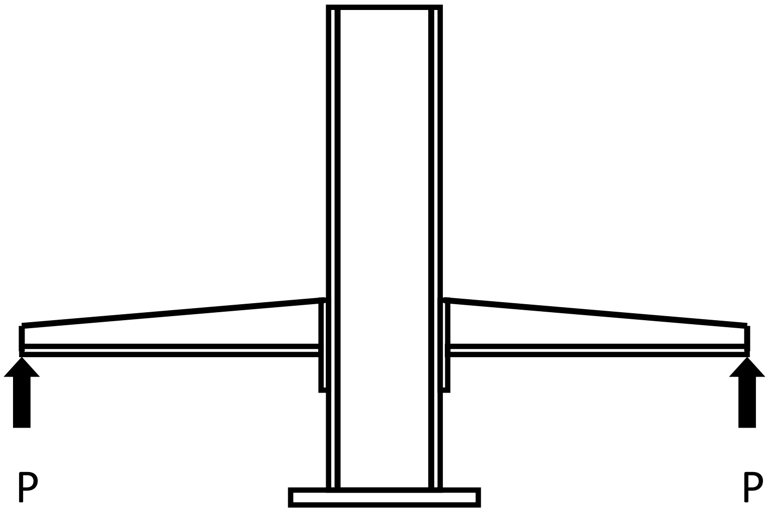


Fig. 8 Test arrangement.

Fig. 9 Test specimen.

5. Results and Discussion

5.1 The Location of the Maximum Bending Stress

It is often assumed that the maximum bending stress of a tapered cantilever lies at the support where the maximum moment occurs. However, due to varying geometric section properties along the length of the beam, the maximum bending stress may not occur at the support and is in fact dependent upon the degree of taper and the applied loading type. When a UDL is applied the maximum stress always occurs at the support, as shown in Fig. 1a, while a point load at the tip is applied, the location of the maximum stress depends on the degree of taper, as shown in Fig. 1b.

It was found that when subjected to a point load at the tip and the degree of taper is greater than two and up to seven, Miller's equation could be used to determine the location of the maximum stress. This could be because a tee section has no bottom flange and only its web is affected by taper. Hence, the cross-section of the tee can be regarded as being rectangular. When the degree of taper is greater than seven, Miller's equation does not accurately predict the location of the maximum stress, as shown in Fig. 4. The difference is as much as 5% when the degree of taper is 12. 5% could be crucial for long beams. The difference could be due to the fact that Miller's equation is based on rectangular cross-sections, while the other two on tee sections. As the degree of taper increases, the cross-sectional area of the web towards to the tip becomes relatively small, compared with that of the flange. Miller's equation ignores the latter. Eq. (8) should be used to determine the location of the maximum stress.

It should be noted that the location of the maximum stress of a tapered tee-section cantilever subject to a point load at the tip is governed by the degree of taper, as shown in Fig. 4. Neither the length of the cantilever nor the magnitude of the point load has an effect upon the location of the maximum compressive stress. Section designations have little influence on the location, as shown in Fig. 10. It should also be noted that Eq. (2) and Fig. 4 are only valid for web tapered tee cantilevers with a point load at the tip and when the support depth is bigger than the tip depth.

The bending stresses of all six beams from the experiment are presented in Fig. 11. As expected, the stress patterns resemble those in Fig. 1b. The maximum stress occurs at the support for the untapered and 2:1 tapered beams, while it moves away from the support for the 3:1 tapered beams. Using Eq. (2) the predicted position, x/L of the maximum stress of the

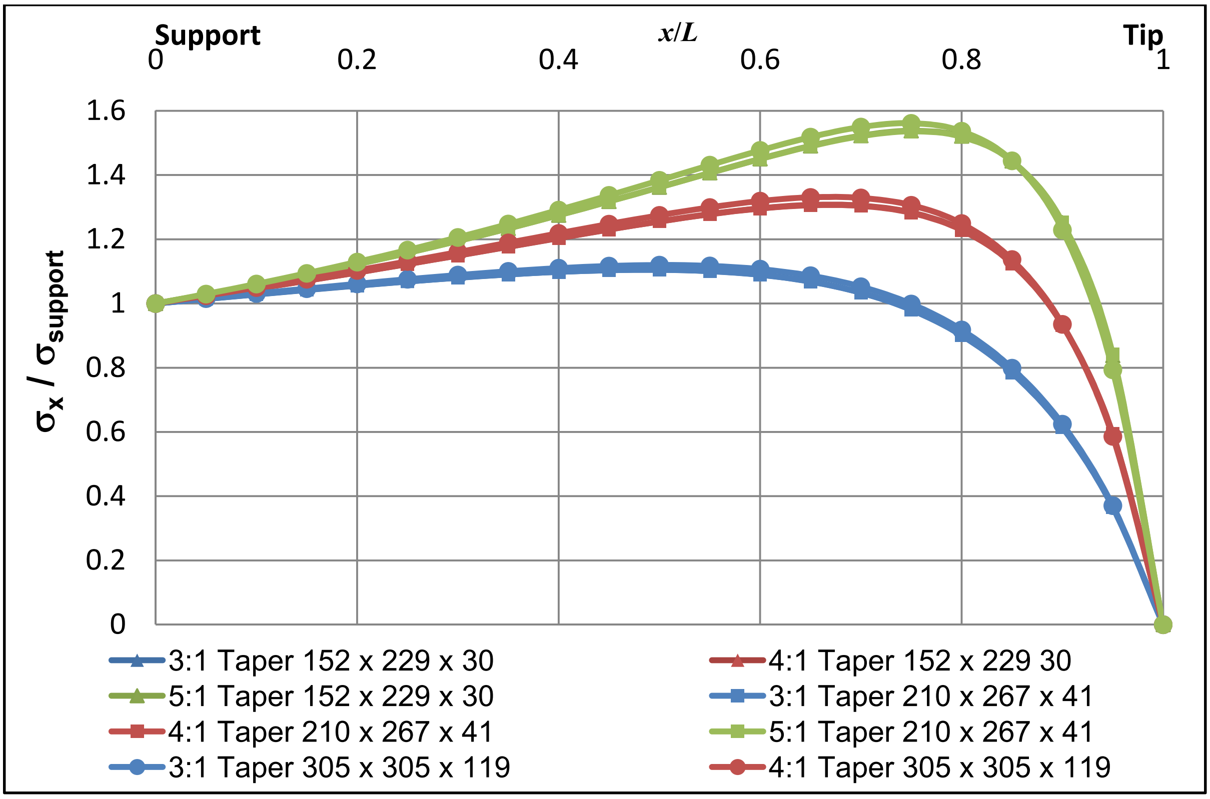
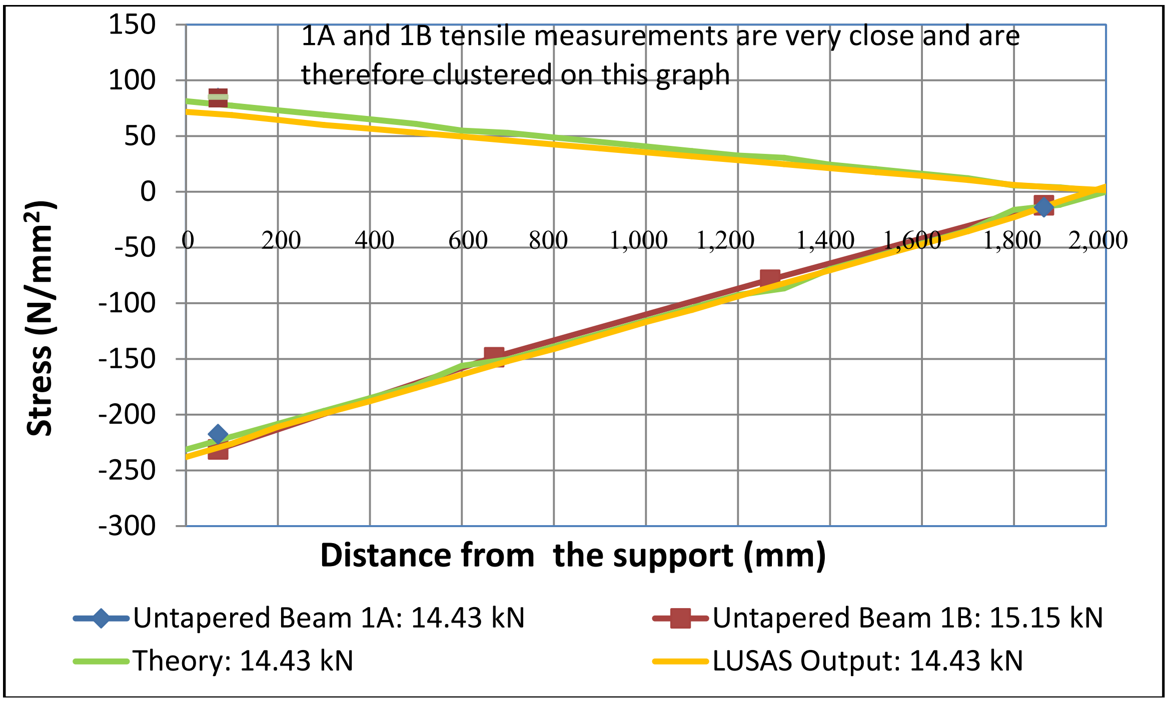
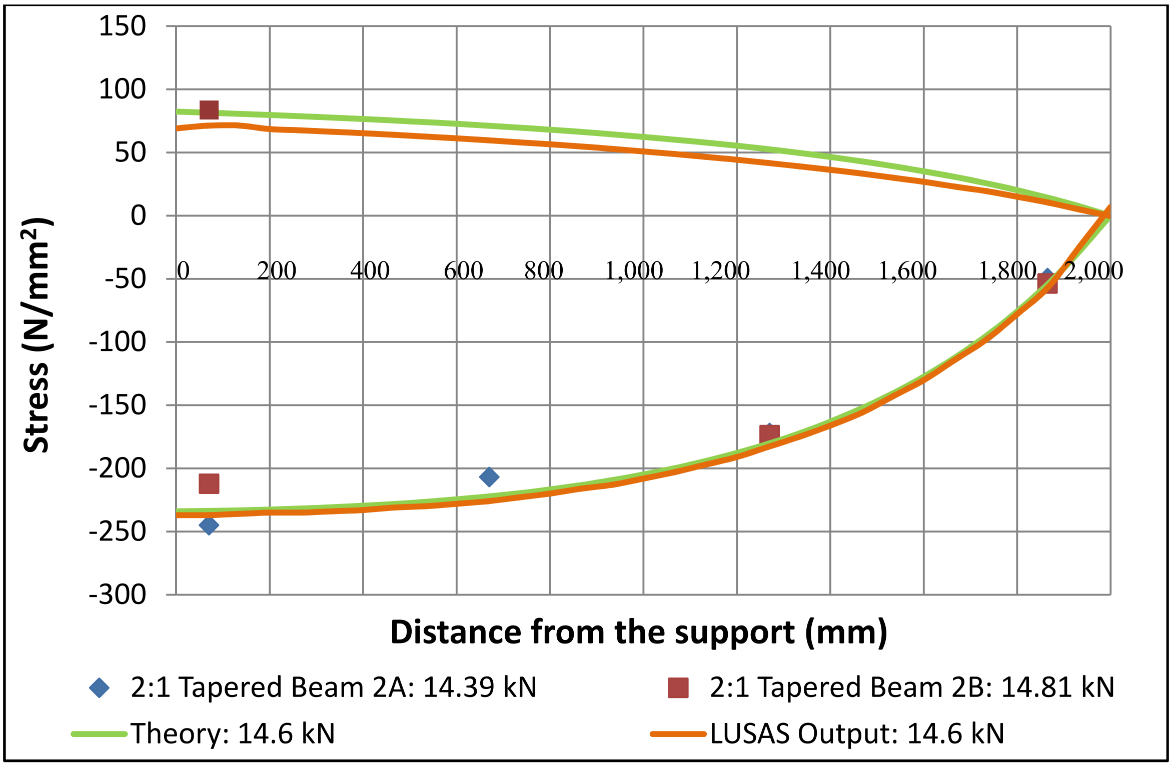


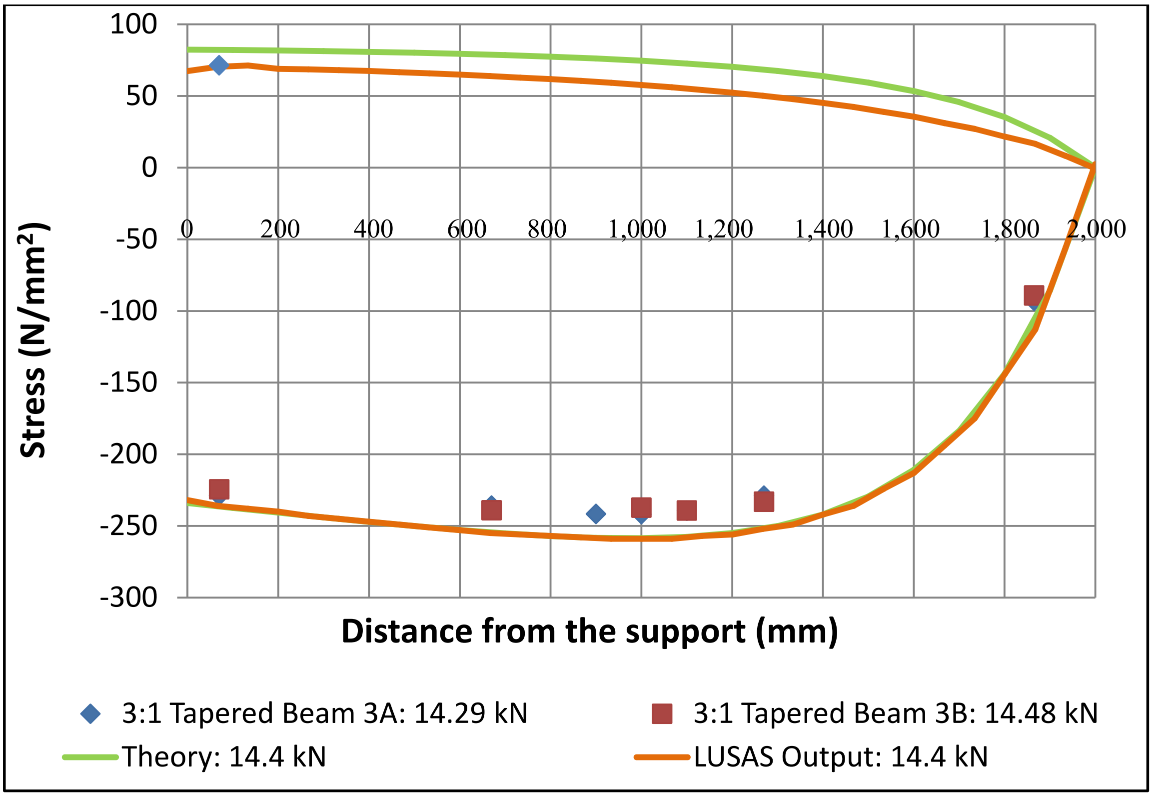
Fig. 10 The locations of the theoretical maximum compressive stress with various tee section designations.



(a) Untapered beam



(b) 2:1 taper



(c) 3:1 taper

Fig. 11 Comparisons of predicted and observed bending stresses along the beam.

Table 3 Comparison between the proposed method with the theory.

Section	254 × 343 × 76	171 × 178 × 26	127 × 178 × 17	102 × 127 × 13	133 × 102 × 15
h (mm)	343.7	177.4	174.4	128.5	103.3
b_f (mm)	254.5	171.5	125.4	101.9	133.9
t_f (mm)	21.0	11.5	8.5	8.4	9.6
t_w (mm)	13.2	7.4	6.0	6.0	6.4
Geometric ratio, β	0.849	0.666	0.982	0.901	0.514
Stress ratio (beam theory)	1.105	1.112	1.096	1.105	1.126
Stress ratio, ψ using Fig. 5	1.104	1.115	1.097	1.100	1.122
Percentage difference (%)	0.049	-0.229	-0.088	0.414	0.386

3:1 tapered beam was at the mid-span (x/L = 0.5), which is 1 m away from the support. Eq. (2) gives a good agreement with this position, as shown in Fig. 11. More tests with cantilevers with higher degrees of taper should be carried out in order to test Eq. (2).

5.2 The Comparison of the Experimental Bending Stresses with the Theory and the FEA

Fig. 11 compares the experimental bending stresses along the beam with the theory and the FEA. The stresses in the web are approximately up to 20 N/mm^2 less than predicted. They agree within a maximum difference of 10%. Part of the difference could be because the beam was inverted and the self-weight of the beam cancelled approximately 4.4% of the applied load for the untapered beam. This could have caused a reduction of 2.2% in the bending moment. Further discrepancies between results could be due to experimental uncertainties in strain gauge measurements and the values of Young's modulus used.

5.3 The Validation of the Method Determining the Magnitude of the Maximum Stress

The proposed method for determining the magnitude of the maximum stress was validated. 5 UK tee sections were randomly chosen. These are not included in Table 2. Table 3 compares the values of ψ using Fig. 5 with stress ratios using the theory for 3:1 tapered beams. The errors of the proposed method are less than 1%.

6. Conclusions

The bending stresses of steel web tapered tee-section cantilevers have been investigated. The cross-sectional properties of such beams vary along the length and therefore the highest stress may not always occur at the support. The bending stresses of such beams depend on the degree of taper and the moment gradient. In design, the bending stresses should be checked not only at the support, but also along the beam. The location of the maximum compressive stress for various degrees of tapered cantilevers was identified. It has been found that when a UDL was applied, the maximum stress always occurs at the support.

When a point load was applied at the tip, it was found that the maximum stress is always at the support for tapered tee cantilevers, whose tapering ratio (h/h') was less than or equal to two. It was also shown that as the degree of taper increased, the location of the maximum stress moved towards the cantilever tip and the magnitude of the stress also increased. It has also been found that Miller's equation (Eq. (2)) can be used to determine the location of the maximum stress of a tapered tee section cantilever whose tapering ratio is greater than two and up to seven. Here, it should be noted that the location of the maximum stress is solely influenced by the degree of taper. When the degree of taper is greater than seven, Miller's equation is not suitable and Eq. (8) should be used to determine the location of the maximum stress. The determination for the location of the maximum stress can be summarized in Eqs. (10)-(13).

When subjected to a UDL:

For all degrees of taper

$$\frac{x}{L} = 0 \tag{10}$$

When subjected to a point load at the tip,

$$\frac{h}{h'} \leq 2 \qquad \frac{x}{L} = 0 \tag{11}$$

$$2 < \frac{h}{h'} \leq 7 \qquad \frac{x}{L} = \frac{\frac{h}{h'} - 2}{\frac{h}{h'} - 1} \tag{12}$$

$$\frac{h}{h'} > 7 \tag{13}$$

for Eq. (13), $\frac{x}{L}$ can be determined using Eq. (8).

A method for determining the magnitude of the maximum bending stress was proposed. It was found that the ratio of the maximum stress to the stress at the support, ψ is determined by the geometric ratio at the support, β, Eq. (9). The proposed chart, Fig. 5 can predict the stress ratio within 1% error.

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