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Energy, Passivity, and Morphological Capacities

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Abstract. The morphology of a system may be exploited to reduce the controller workload or to perform specific tasks, e.g., locomotion processes. However, understanding and characterizing these capacities in terms of physical quantities remains an open problem. In this paper, we pose the question: *are energy and dissipation useful physical quantities to characterize morphological capacities?* First, to illustrate the relevance of this question, we present some basic concepts of passivity theory that are useful for carrying out an energy-based analysis of the behavior of dynamical systems. Then, we analyze the behavior of a mechanical system, and we show how its morphology affects its energy. Finally, we present a detailed analysis of a passive walker and explain how its locomotion capacities can be interpreted in terms of energy.

1. Introduction

The relevance of the morphology in biological systems has been studied for a long time (see, for instance, the discussion provided in [1, 2]). Similarly, there is a vast range of applications where the morphology of systems has been engineered for diverse purposes. For instance, the design of soft robots with different shapes and made of different materials is a current hot-topic in robotics [3, 4]. Regarding art, the *Strandbeests* created by Theo Jansen (see [5]) illustrate how to exploit the morphology for locomotion purposes. In particular, the morphology of an embodied system may exhibit capacities to perform tasks often associated with the system's brain (control). Depending on the extent of such capacities, this phenomenon can be understood as a degree of *embodied intelligence*. Examples of bodies performing tasks with minimal—or without—intervention of the brain are abundant in nature, e.g., an octopus arm, the whiskers of a cat, and the muscles of our legs, while passive walkers are a celebrated example in robotics [6]. More examples of tasks performed by the morphology of a system are mentioned in detail in [7, 8].

Several terms have been coined to describe how and when the morphology of a system reduces its brain's workload. Among the proposed terminology, the concept of *morphological computation* has been thoroughly described and explained, e.g., [8, 9]. Notably, a great effort has been made to unify concepts and results regarding the relationship between the morphology and intelligence of physical systems. Nevertheless, several questions remain to be addressed in this field. In particular, how to intuitively characterize the morphological capacities of an embodied system in terms of physical quantities. This question is the cornerstone for the design, analysis, and control of embodied systems in an efficient way.

A thorough exposition on morphological capacities is provided in [7], where the authors recurrently underscore how energy storage and dissipation affect an embodied system. Interestingly, the concepts of energy and dissipation have been used in fields such as mechanics, dynamical systems, and control theory to study the behavior of complex systems. Following this rationale, in this paper we pose the question: *are energy and dissipation useful physical quantities to characterize morphological capacities?* To study this question formally, we use passivity theory, which has roots in electrical circuits theory but has additionally proven extremely useful in control systems [10, 11] and robotics [12, 13].

While there exists literature where morphological capacities are quantified and mathematically formalized, e.g., [14, 15, 16], here we propose for the first time—to the best of our knowledge—passivity, energy, and dissipation as concepts suitable to understand, characterize, and formalize the morphological capacities of an embodied system. In particular, our goal is to pave the way to develop an alternative framework to analyze such capacities intuitively. However, we stress that such an alternative is not in conflict with the existing results in this field. On the contrary, it may represent an excellent complement to them and, in this way, contribute towards a unified framework for the study of morphological capacities and their relationship with embodied intelligence.

2. Preliminaries

2.1. Notation

The symbol $\mathbf{0}$ represents a vector or matrix whose entries are zeros. The ith element of the vector $x \in \mathbb{R}^n$ is denoted by x_i. Consider a scalar multivariable function $f : \mathbb{R}^n \to \mathbb{R}^n$. We adopt the convention

$$\frac{\partial f(x)}{\partial x} = \begin{bmatrix} \dfrac{\partial f(x)}{\partial x_1} \\ \vdots \\ \dfrac{\partial f(x)}{\partial x_n} \end{bmatrix}.$$

We use Newton's notation for differentiation with respect to time, i.e.,

$$\dot{x} = \frac{dx(t)}{dt}.$$

The symbols e and sgn represent the exponential and signum functions, respectively.

2.2. Morphological Capacities

The term morphological computation is commonly used to refer to the capacity of the body to perform tasks without the intervention of the brain in an embodied system. However, we remark that the definition of the mentioned term can vary from one reference to another, where a fundamental difference lies in defining when systems compute. For instance, in [17], the authors understand morphological computation as the phenomenon of morphology and material taking over some of the functions often attributed to the brain of a system. However, the mentioned definition does not meet the requirements established in [8], where the authors define three roles of morphology: morphology facilitating control, morphology facilitating perception, and morphological computation. We refer the reader interested in this discussion to [7, 18, 19, 9] and the previous references.

To avoid controversy, we use the term morphological capacities to indicate that: (a) the morphology permits performing specific tasks without requiring a control input (brain action), or (b) the morphology simplifies the control signal (brain's action) required to perform a task. In particular, we focus on tasks related to locomotion. However, the ideas exposed in this paper can be adapted to perception tasks. For example, in [20], the authors use a similar analysis to detect collision in a robotic arm. Moreover, the characterization of morphological capacities presented in this paper can be adapted to the different definitions of morphological computation and other roles of morphology.

2.3. Passivity

In this section, we revisit some passivity theory concepts. In particular, we restrict our attention to input-affine nonlinear systems in state-space representation for two reasons: (i) these systems are convenient to illustrate the main idea of this manuscript, and (ii) we avoid unnecessarily dense mathematical content. However, the reader interested in general dissipativity and passivity theory is referred to the seminal works [21, 22] and the more recent references [23, 10].

The concept of passivity has its roots in circuit theory. However, it has proven extremely useful for analyzing and controlling dynamical systems, particularly physical ones. Passivity is an input-output property, where, informally, we consider that a system is passive if it cannot generate energy. To formalize this idea, we consider a system, whose dynamics are described by

$$\dot{x} = f(x(t)) + g(x(t))u(t), \tag{1}$$

where $t \in \mathbb{R}_{\geq 0}$ denotes time, $x : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ represents the states of the system; $u : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ denotes the input vector, with $m \leq n$; $f : \mathbb{R}^n \to \mathbb{R}^n$; and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$. Additionally, we consider that the output of (1) is a signal of dimension m, given by

$$y(t) = h(x(t)), \tag{2}$$

where $h : \mathbb{R}^n \to \mathbb{R}^m$.

Throughout this paper, we assume that the solution $x(t)$ of (1) is unique; the signals $u(t)$ and $y(t)$ are bounded, and the integral $\int_0^t u^\top(\tau)y(\tau)d\tau$ is well-defined.

Definition 1 (Passive system) *A system of the form* (1) *is said to be passive if there exists a function* $S : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ *such that*

$$S(x(t)) \leq S(x(0)) + \int_0^t u^\top(\tau)y(\tau)d\tau, \qquad \forall\ t \in \mathbb{R}_{\geq 0}. \tag{3}$$

In passivity theory jargon, $S(x(t))$ is referred to as the *storage function* and the inequality (3) is known as the *dissipation inequality*. Moreover, if (3) holds, $y(t)$ is referred to as the *passive output* of (1).

For our purposes, we consider that the storage function of a physical system denotes its energy. Furthermore, the units of $u(t)$ and $y(t)$ are such that their inner product has power units, i.e., $u^\top(t)y(t)$ represents the power injected into the system. Thus, (3) implies that the energy of the system (1) at t cannot be greater than the energy previously stored by the system, plus the energy injected during the interval $[0, t]$. Henceforth, to ease the readability, we omit the argument t from the state x and the signals u and y.

Note that, under some differentiability assumptions, we can rewrite (3) as

$$\dot{S} \leq u^\top y, \tag{4}$$

which is referred to as the *differential dissipation inequality*. In this case, (4) implies that the power that can be extracted from the system at t should be less than or equal to the power injected into the system.

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Definition 2 (Lossless system) *A system of the form* (1) *is said to be lossless if* (3)*—alternatively,* (4)*—holds with equality.*

Note that the energy of a lossless system, at t, equals the sum of the stored energy and the energy injected into the system during the interval $[0, t]$. In this regard, every lossless system is passive, but the converse is not true.

Passivity theory is not limited to linear systems. Moreover, a broad range of physical systems is passive. Therefore, the arguments exposed above are suitable to analyze the behavior of a wide variety of systems even if their dynamics are highly complex, e.g., nonlinear systems.

3. Energy as the Orchestra's Director

Intuitively, the behavior of a physical system is ruled by its energy. Consequently, energy-based modeling frameworks have been extensively used to describe the behavior of physical system, e.g., the Euler-Lagrange formalism [24] and the port-Hamiltonian framework [25, 26].

To illustrate the role of energy in the behavior of a physical system, let us consider the system depicted in Fig. 1, which consists of two pendulums moving along the horizontal plane, where the first pendulum is attached to the reference through an ideal torsion spring and the second pendulum can rotate freely. Additionally, m_1 and m_2 represent the mass of the first and second pendulum, respectively; ℓ_1 and ℓ_2 denote the length of the first and second pendulum, respectively; and k represents the spring constant.

A suitable choice for the state vector is

$$x = \begin{bmatrix} q \\ p \end{bmatrix}; \qquad \begin{array}{c} q \in \mathbb{R}^2 \\ p \in \mathbb{R}^2, \end{array} \tag{5}$$

where q represents the angular position of the pendulums and p the corresponding momenta. For simplicity, we consider that the rods are rigid, massless, and inextensible; moreover, we assume that the masses at the tip of each pendulum cannot rotate on their own axis. Under these assumptions, the expression that characterizes the potential energy is

$$V(q) = \frac{1}{2} k q_1^2, \tag{6}$$

whereas the kinetic energy is given by

$$T(q,p) = \frac{1}{2} p^\top M^{-1}(q) p; \qquad M(q) := \begin{bmatrix} \ell_1^2(m_1 + m_2) & \ell_1 \ell_2 m_2 \cos(q_1 - q_2) \\ \ell_1 \ell_2 m_2 \cos(q_1 - q_2) & \ell_2^2 m_2 \end{bmatrix}. \tag{7}$$

Hence, the total energy of the system—i.e., the system's Hamitonian—is

$$H(q,p) = T(q,p) + V(q). \tag{8}$$

Moreover, the dynamics of the system are described by

$$\begin{aligned} \dot{q} &= \frac{\partial H(q,p)}{\partial p}, \\ \dot{p} &= -\frac{\partial H(q,p)}{\partial q}. \end{aligned} \tag{9}$$

Note that, given the choice of state variables q and p, the dynamics of the system are completely determined by its energy. At this point, we make the following remarks:

- This system only stores *elastic* potential energy because of the spring. Consequently, the spring exerts a force on the first link when this is not aligned with the Y-axis.
- Because we are not considering friction or dampers, there is no dissipation, i.e., this system is lossless.

Observing the schematic provided in Fig. 1, we can obtain the configuration of the system in the cartesian plane via the following expressions:

$$\begin{array}{rclcrcl} X_{\mathtt{m}_1} & = & \ell_1 \sin(q_1), & X_{\mathtt{m}_2} & = & \ell_1 \sin(q_1) + \ell_2 \sin(q_2), \\ Y_{\mathtt{m}_1} & = & \ell_1 \cos(q_1), & Y_{\mathtt{m}_2} & = & \ell_1 \cos(q_1) + \ell_2 \cos(q_2). \end{array} \tag{10}$$

Fig. 1 depicts the behavior of this system in cartesian coordinates for different initial conditions of the second link, where we consider $\ell_1 = 2$ [m], $\ell_2 = 0.5$ [m], $m_1 = 1$ [kg], $m_2 = 2.5$ [kg], and $k = 1$ [N/m] for simulations purposes. In the three cases, we consider that the initial conditions of the momenta are zero, i.e., $p_0 = \mathbf{0}$. Figs. 1 and 2 show that different initial conditions of the second link yield completely different trajectories, while the first link describes the same semi-circular trajectory for the three cases. Such behavior could seem random. Nevertheless, it is entirely characterized by the energy of the system. To note this, we recall that the system is lossless. Thus,

$$\dot{H} = 0 \implies \int_0^t \dot{H} dt = 0 \implies H(q(t), p(t)) = H(q_0, p_0); \ \forall \ t \in \mathbb{R}_{\geq 0}, \tag{11}$$

where $q_0 = q(0)$ and $p_0 = p(0)$. Therefore, because we consider $p_0 = \mathbf{0}$, the expression that describes the system's energy reduces to

$$H(q(t), p(t)) = \frac{1}{2} k q_1^2(0), \ \forall \ t \in \mathbb{R}_{\geq 0}. \tag{12}$$

Consequently, (12) implies that the trajectories of the system must guarantee that the energy remains constant all the time.

We consider the same initial condition $q_1(0)$ in the three proposed simulations. Hence, the total energy should be the same even if the kinetic and potential energies evolve differently for each set of initial conditions, which is corroborated in Fig. 2.

Suppose that the system consists only of the links, and the spring is an external body. Under this hypothesis, the energy of the system is given only by $T(q, p)$. Moreover, the dynamics of the system are

$$\begin{array}{rcl} \dot{q} & = & \dfrac{\partial T(q,p)}{\partial p}, \\ \dot{p} & = & -\dfrac{\partial T(q,p)}{\partial q} - A F_{\mathtt{k}}(q_1), \end{array} \tag{13}$$

where

$$F_{\mathtt{k}}(q_1) = k q_1, \qquad A := \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{14}$$

Hence, some simple computations, omitted for brevity, yield

$$\dot{T} = F_{\mathtt{k}}(q_1) \dot{q}_1, \tag{15}$$

where the passive output is given by $\dot{q}_1$. Note that the system is still lossless, but we consider the force $F_{\mathtt{k}}(q_1)$ as an external input. Therefore,

$$p_0 = \mathbf{0} \implies T(q_0, p_0) = 0. \tag{16}$$

Accordingly, the (kinetic) energy varies only if the supply rate $F_{\mathtt{k}}(q_1)\dot{q}_1$ is different from zero. The physical interpretation of this analysis is that the system moves only if the spring exerts a force on the first link, i.e., some power is injected into the system.

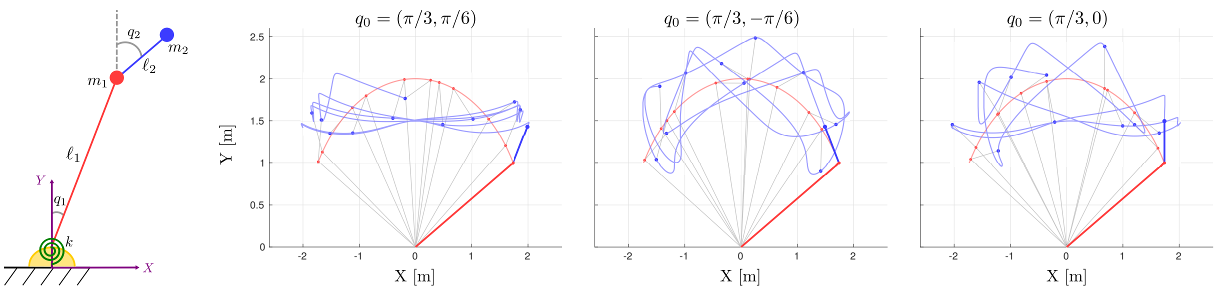

Figure 1. **(Left)** Schematic of the two-links system moving along the horizontal plane. **(Right)** Trajectories of the system under different initial conditions, denoted as q_0. The initial configuration is indicated with thicker lines.

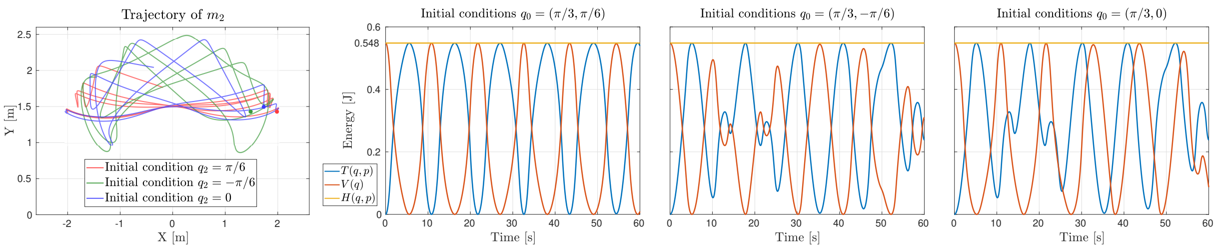

Figure 2. **(Left)** Evolution of m_2 under different initial conditions for q_2. **(Right)** Evolution of the potential, kinetic, and total energy of the system for different initial conditions.

3.1. Dissipation and equilibria

Dissipation plays an essential role in the stabilization of mechanical systems. Loosely speaking, dissipative elements (e.g., dampers and friction) make the system lose energy until it eventually reaches *a* minimum energy value. Such a phenomenon is related to the concept of equilibrium points, namely, points (configurations) where the system stays if there are no external inputs. Note that a configuration is an equilibrium point only the dynamics of the system are zero at that point. Thus, intuitively, equilibrium points are characterized by the critical points of the system's Hamiltonian (energy). Moreover, because the kinetic energy is quadratic in p, the equilibrium points of a mechanical system are given by points of the form $(\bar{q}, \mathbf{0})$, where $\bar{q} \in \mathbb{R}^n$ is a critical point of the potential energy.

Consider the two-links system, depicted in Fig. 1, and suppose there exists friction between the second link and a surface. The dynamics of the system become[1]

$$\begin{aligned} \dot{q} &= \frac{\partial H(q,p)}{\partial p}, \\ \dot{p} &= -\frac{\partial H(q,p)}{\partial q} - AF_{\mathfrak{f}}(\dot{q}_1), \end{aligned} \tag{17}$$

where $H(q,p)$ and A are defined in (8) and (14), respectively, and

$$F_{\mathfrak{f}}(\dot{q}_1)\dot{q}_1 > 0, \ \forall \ \dot{q}_1 \neq 0. \tag{18}$$

The critical points of the energy of this system are characterized by $q_1 = 0$ and $p = \mathbf{0}$. Therefore, this system has an infinite number of equilibrium points. In particular, any point of the form

[1] Note that $\dot{q}$ is not a state of the system in our analysis. However, we introduce the abuse of notation $F_{\mathfrak{f}}(\dot{q}_1)$ to preserve the physical intuition of this force.

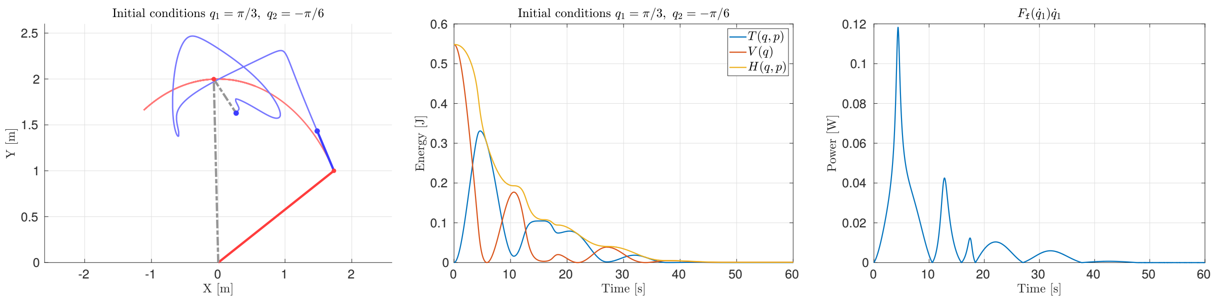

Figure 3. **(Left)** Trajectories of the system. The initial configuration is depicted with solid lines, while the configuration after 60 seconds is plotted with dashed lines. **(Middle)** Evolution of the potential, kinetic, and total energy of the system. **(Right)** Supply rate associated with friction.

$(0, q_2, 0, 0)$ is an equilibrium for the system.

If we consider the force $F_\mathfrak{f}(\dot{q}_1)$ as an external input, then the system (17) is passive, where the passive output is given by $\dot{q}_1$. In particular, we have that

$$\dot{H} = -F_\mathfrak{f}(\dot{q}_1)\dot{q}_1. \tag{19}$$

Therefore, from (18), the supply rate is negative for $\dot{q}_1$ different from zero. In other words, friction implies that power is extracted from the system until the second link stops moving. Fig. 3 shows the simulation of the behavior of (17), where, inspired by [27], we consider the friction model

$$F_\mathfrak{f}(\dot{q}_1) = \left[b_1 \mathrm{e}^{-(\dot{q}_1/b_2)^2} + b_3\right] \mathrm{sgn}(\dot{q}_1) + b_4\dot{q}_1, \tag{20}$$

with $b_1 = 0.05$, $b_2 = 0.5$, $b_3 = 0.01$, and $b_4 = 0.8$. In the mentioned figure, we notice that the first link converges to a configuration close to $q_1 = 0$. Note that the first link does not stabilize exactly at zero because of the signum function in (20), which is related to dry friction. Moreover, the trajectories of the second link are different than the ones shown in Fig. 1 under the same initial conditions. Thus, we conclude that the presence of friction in the first link also affects the behavior of the second link. Also in Fig. 3, we observe that the stored energy converges to zero and that the power related to friction decreases as the system's energy decreases. Note that this supply rate represents the amount of energy extracted from the system. Hence, it is expected to become zero when the system has no stored energy. Moreover, because friction dissipates energy, the supply rate should not be negative,[2] implying that some energy is injected into the system. Both properties can be corroborated in Fig. 3.

3.2. Other external inputs

As explained in Subsection 3.1, the effect of dissipation can be understood as a supply rate that extracts power from the system. Note that this supply rate results from the interaction of the system with its environment. However, supply rates may originate from signals that could exist independently of the body of the system, i.e., signals that are independent of the system's dynamics. Examples of these signals are control inputs. These supply rates can drastically affect the energy and, thus, the natural behavior of the system. To illustrate this, we consider the two-links system neglecting any dissipation but supposing there is an external force $u : \mathbb{R}_{\geq 0} \to \mathbb{R}$ acting on the first link. Hence, the dynamics of the system become

[2] Notice the minus sign in (19).

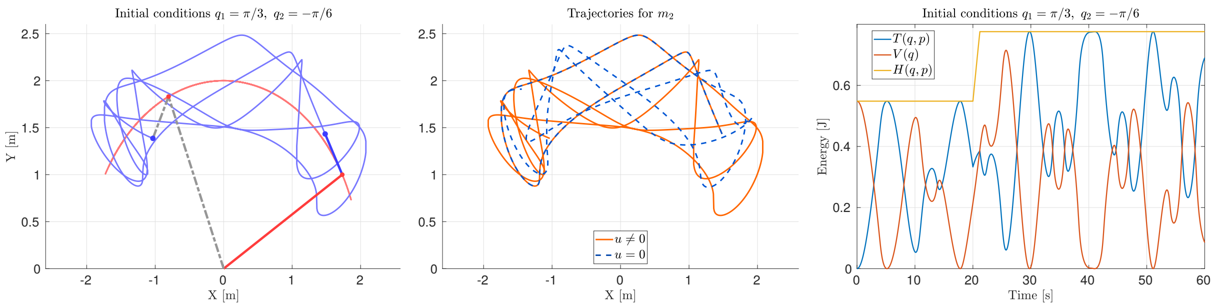

Figure 4. **(Left)** Trajectories of the system. The initial configuration is depicted with solid lines, while the configuration after 60 seconds is plotted with dashed lines. **(Middle)** Evolution of m_2 under the same initial conditions with and without external input. **(Right)** Evolution of the potential, kinetic, and total energy of the system.

$$\begin{aligned} \dot{q} &= \frac{\partial H(q,p)}{\partial p}, \\ \dot{p} &= -\frac{\partial H(q,p)}{\partial q} + Au(t). \end{aligned} \tag{21}$$

We consider that $u(t)$ is a unitary pulse starting at $t = 20$ and lasting 1.2 seconds for illustration purposes. Fig. 4 shows the simulations of this system, where we observe the following:

(Left plot). The oscillations of the first link exceed the initial conditions. In particular, the light red curve that describe the trajectory of the first mass overcomes the initial conditions depicted with a solid red dot.

(Middle plot). The external input also affects the trajectory of the second mass. In this plot, both trajectories are identical until the external input modifies the behavior of the system.

(Right plot). At $t = 20$ [s], the energy stored in the system increases because of the power injected by the external input.

Remark 1 *External inputs play a crucial role in control systems, where these signals are designed to guarantee that the system stabilizes at a desired point or follows a desired trajectory. Customarily, these problems are referred to as set-point regulation and trajectory tracking, respectively. Regarding nonlinear control techniques, some passivity-based control strategies consist in designing a control signal that modifies the energy such that the desired equilibrium point becomes a minimum of the energy function. Then, another control input injects damping into the system to ensure its convergence to the desired equilibrium, i.e., the minimum of the closed-loop energy.*

4. Understanding Morphological Capacities in Terms of Energy

The morphology of a system is closely related to its energy, e.g., the potential energy captures any compliant behavior in a mechanical system in the form of elastic potential energy. Also the forces resulting from gravity—interaction with the environment—are characterized by the potential energy. Two common strategies to model mechanical systems are the Euler-Lagrange formalism (see [24]), where the equations of motion are given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D(\dot{q})\dot{q} + \frac{\partial V(q)}{\partial q} = A(q)u(t)$$

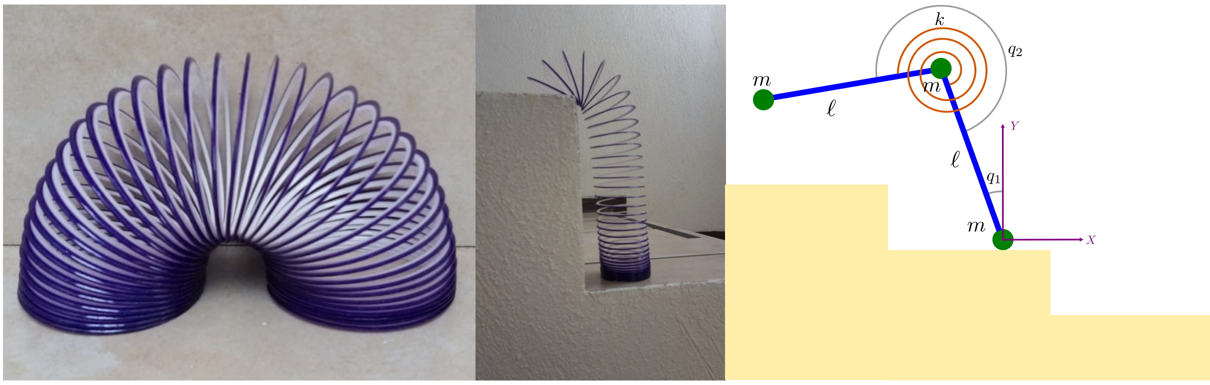

Figure 5. (**Left**) Slinky toy. **(Middle)** Slinky toy walking down some stairs. **(Right)** Simplified schematic of the slinky toy walking down stairs.

and the port-Hamiltonian approach (see [25, 26]), where the dynamics of the system are given by

$$\begin{aligned}\dot{q} &= \frac{\partial H(q,p)}{\partial p},\\ \dot{p} &= -\frac{\partial H(q,p)}{\partial q} - D(q,p)\frac{\partial H(q,p)}{\partial p} + A(q)u(t).\end{aligned}$$

Note that in both cases some simple computations lead to the conclusion that the equilibrium of a mechanical system is given by $(q_\star, \mathbf{0})$, where $q_\star \in \mathbb{R}^n$ is determined by the values of the configuration variables q such that

$$\frac{\partial V(q)}{\partial q} = \mathbf{0}.$$

Consequently, if there are no external inputs—i.e., $u(t) = \mathbf{0}$—any mechanical system with dissipation converges to a critical point of its potential energy. Hence, modifying the morphology of a system determines where it naturally stabilizes. An illustrative example is the Gömböc, a mono-static body whose potential energy has only two equilibria: one stable and the other unstable. Accordingly, whenever the system is placed in an initial condition different from the unstable equilibrium, it self-stabilizes at the stable equilibrium. A similar principle is exhibited by the shape of some turtles' shells that can self-right themselves with minimum effort. For more discussion on this topic, we refer the reader to [28, 29].

An example of how morphology can facilitate locomotion is given by slinky toys, such as the one depicted in Fig. 5. These systems have some interesting properties due to their morphology, e.g., [30]. In particular, their compliant nature lets them move downstairs without any external (control) input involved. In this section, inspired by a slinky toy, we propose the mechanical system presented in the right image of Fig. 5, which consists of three identical masses, two identical rods, and a torsion spring. Then, we model how the system descends each step. To this end, we modify the model provided in [31] by changing the references to illustrate more transparently the role of energy in locomotion. The resulting model is of the form provided in (9), with

$$M = \ell^2 \begin{bmatrix} 2 & -\cos(q_1 - q_2) \\ -\cos(q_1 - q_2) & 1 \end{bmatrix}, \quad V(q) = \frac{1}{2}k(q_1 + q_2)^2 + mg_{\mathrm{r}}\ell\left(\cos(q_1) - \cos(q_2)\right). \tag{22}$$

Note that q_1 corresponds to the angular position of middle mass, while q_2 denotes the angular position of the mass at the tip of the rear leg (rod). However, once the system walks down a

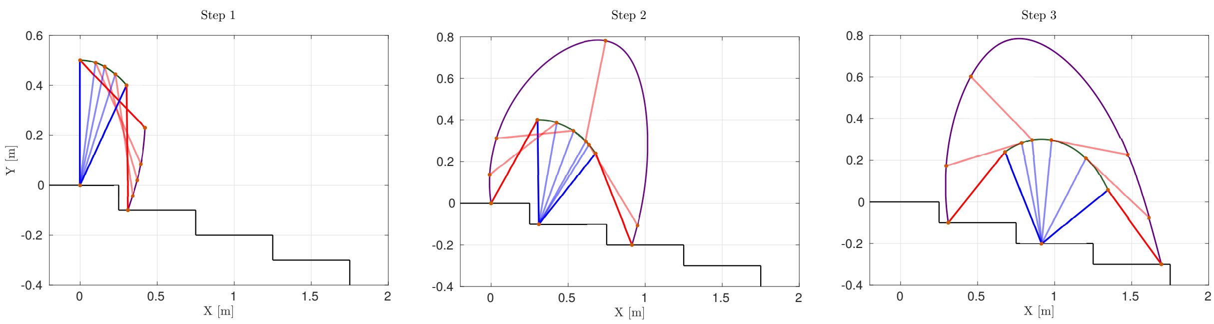

Figure 6. Motion of the slinky-like system while walking down the steps. The rear and front legs are depicted in red and blue, respectively. Moreover, the initial and final configuration of each step are presented in solid lines. Additionally, we show the trajectories of q_1 and q_2 in green and purple, respectively.

step, the front leg becomes the rear one and vice-versa. For simulation purposes, we consider three steps 0.1 [m] height and 0.5 [m] of width. Moreover, the values of the parameters used for simulation are: $\ell = 0.5$ [m], $m = 0.1$ [kg], and $k = 0.5$ [N/m]. Fig. 6 depicts the evolution of the system while walking down the three steps. Moreover, the left image of Fig. 7 shows the initial condition of the system and its position when making contact with each step. We observe that the angle between both legs increases with each step. Hence, its (standalone) locomotion capacity is limited. However, it can perform a task relying only on its morphology—particularly the torsion spring—and its interaction with the environment—the effect of gravity. We stress that we neglect any dissipation phenomenon. Therefore, the process is lossless, as is shown in the middle plot of Fig. 7. Similarly to the two-links example in Section 3, we can consider the force of exerted by the spring as an external force to analyze the locomotion process. Thus, the dynamics take the form (13), with

$$F_{\mathtt{k}}(q) = k(q_1 + q_2), \qquad A = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{23}$$

Moreover, the differential dissipation inequality takes the form

$$\dot{H} = -\dot{q}^\top A F_{\mathtt{k}}(q) = -k(q_1 + q_2)\,(\dot{q}_1 + \dot{q}_2) \tag{24}$$

The right plot of Fig. 7 shows the profile of the power (supply rate) associated with the spring. This supply rate can be interpreted as a control signal commanded by the body instead of the brain, i.e., the morphology has the capacity to generate control inputs to perform a specific task. While the system's ability to learn or adapt is limited in this example, the proposed analysis can be extended to more complex morphologies.

In [15], the authors highlight the overlapping of morphological computation with control theory, and in [32], the authors borrow concepts from control systems to analyze the importance of feedback in morphological computation. Notably, both control systems and morphological computation are closely related to the dynamics of the system to be studied. As mentioned in Section 3, such dynamics are mainly determined by energy, dissipation, and external inputs. Moreover, even if the body of a system cannot perform a task by itself, it may reduce the control effort. Examples of this phenomenon are the bipedal robots designed in [33], and the locomotion approach proposed in [34].

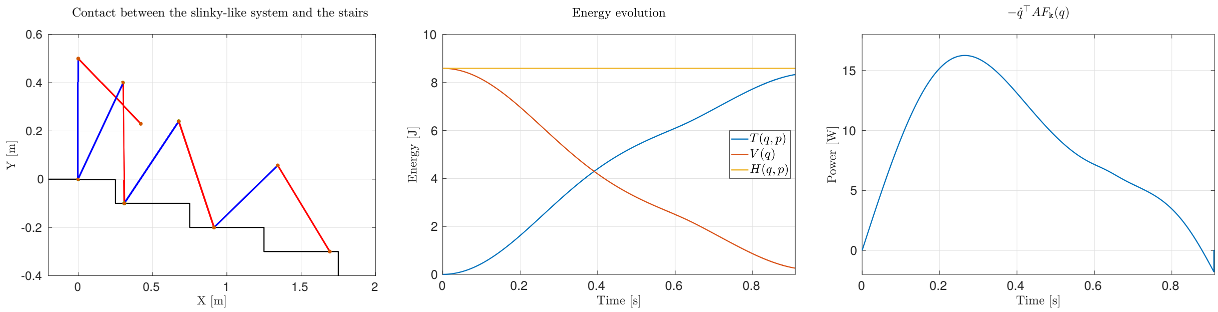

Figure 7. (**Left**) Initial configuration and contact postures of the slinky-like system. The rear and front legs are plotted in red and blue, respectively. (**Middle**) Evolution of the potential, kinetic, and total energy of the system corresponding to the locomotion process from step 1 to step 2. **(Right)** Power related to the torsion spring during the locomotion process from step 1 to step 2.

5. Concluding Remarks and Future Work

Analyzing the energy of a system provides relevant information about its behavior. The morphology of a system determines how energy is dissipated and internally stored in a system. Hence, through an energy-based analysis, we can determine how morphology affects some capacities of the system—e.g., locomotion capacities—while providing a physically-intuitive interpretation of the process. In this paper, we have illustrated how to use some basic concepts of passivity theory to interpret locomotion processes originated by the morphology of a system. However, this is just a humble example of what could be achieved by using energy-based analyses to understand and characterize morphological capacities.

As future work, we envisioned the use of energy-based methods to understand, quantify, and characterize the morphological computation of more complex systems, e.g., systems that exhibit physical reservoir computing (see [8]). We stress that passivity theory only requires information about the system's dynamics and energy to provide a physically intuitive analysis of its behavior. Hence, this theoretical framework may be the cornerstone that complements, unifies, and enlarges current methodologies that quantify and study morphological capacities, such as locomotion.

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